The Mechanics of
Tractor – Implement Performance

Theory and Worked Examples

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CHAPTER 6

HITCHING AND MECHANICS OF THE TRACTOR CHASSIS

CONTENTS

Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.

CHAPTER 6

HITCHING AND MECHANICS OF THE TRACTOR CHASSIS

6.1 INTRODUCTION

It was shown in Chapters 4 and 5 how the weight on the wheels of a tractor determines its tractive force and rolling resistance, hence its drawbar pull and tractive efficiency.

This weight depends on:

(i) the static forces, viz,
* the weight of the tractor
* that part of the implement weight (if any) that is carried by the tractor

(ii) the effect on the tractor of the dynamic forces arising from the action of the implement, viz,
* draught (horizontal) force(s)
* vertical force(s)

In designing and using the tractor - implement system, it is desirable to take advantage of all these forces to increase (and control) the weight on the tractor wheels while still ensuring the satisfactory performance of the tractor and the implement. For a given optimum weight on the wheels, the more that is provided by the dynamic effects, the less that has to be provided by the static weight. The three-point linkage system introduced by Ferguson, which made significant use of the dynamic forces on the implement to provide weight on the driving wheels, allowed the introduction of a very light tractor. This feature is now used on most small to medium sized tractors.

Before considering the mechanics of the tractor chassis we need to review the methods of hitching (attaching) implements to the tractor as these have a significant influence on how the implement forces determine the dynamic weight on the tractor wheels. The following gives a brief review of those aspects of implement hitching that are relevant to the performance of the tractor. Other details of the various systems may be found in the references at the end of this Chapter.

6.2 IMPLEMENT HITCHING

6.2.1 Introduction

The hitching of implements and the mechanics of the chassis may be studied by considering two perpendicular planes:

(i) the vertical longitudinal plane down the centre line of the tractor in which we consider the symmetrical forces such as the weight , the wheel reactions and the direct effect of the implement forces.

(ii) the horizontal plane where the moment effect of the implement forces which are not symmetrical (eg, unsymmetrical or off-set implements and all draft forces in turning) will affect the attitude and steering of the tractor. These influence the operation of the tractor but are not relevant to the normal (straight ahead) performance of the tractor; they will not be considered further in this book..

The hitching of implements to tractors may be made in various ways and places. For this purpose the tractor has one or more standard attachment locations at the rear and for some tractors at the front, in the form of:

(i) linkages for 'adjustable' attachment; adjustment in the vertical plane is usually made by means of an in-built hydraulic (hydro-static) pump driven by the tractor engine.

(ii) drawbars for 'fixed' attachment; adjustment is made manually or with 'external' or 'remote' hydraulic cylinders supplied with oil from the in-built hydraulic pump in the tractor.

The standard hitching systems may be classified as follows.

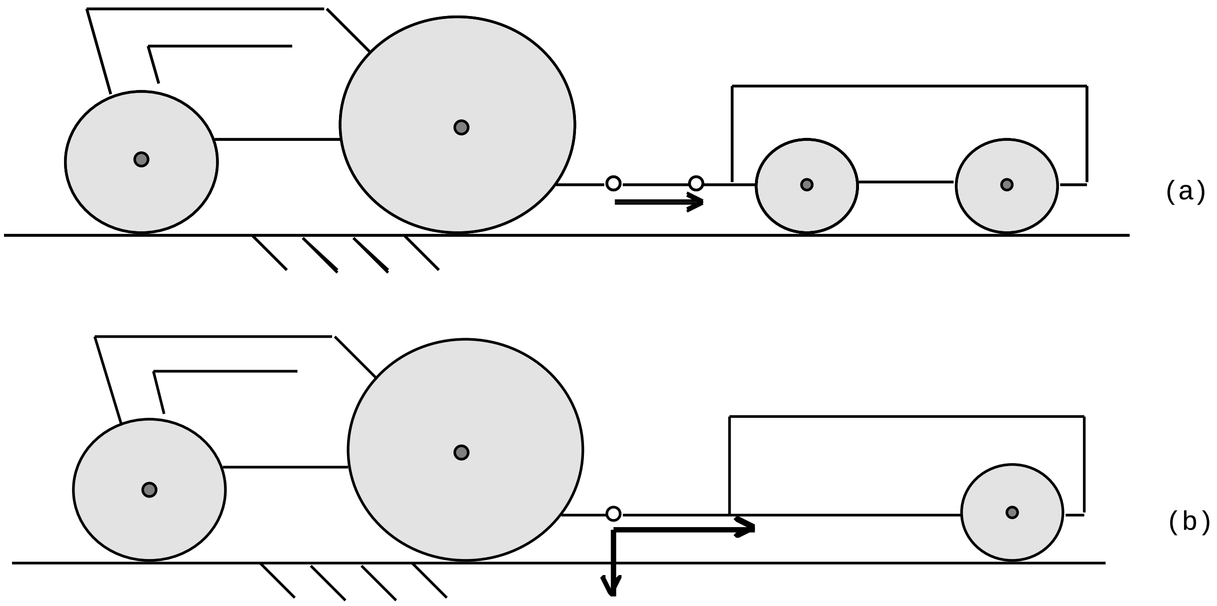


Figure 6.1: Trailed (one point) implement hitches (a) without and (b) with vertical force.

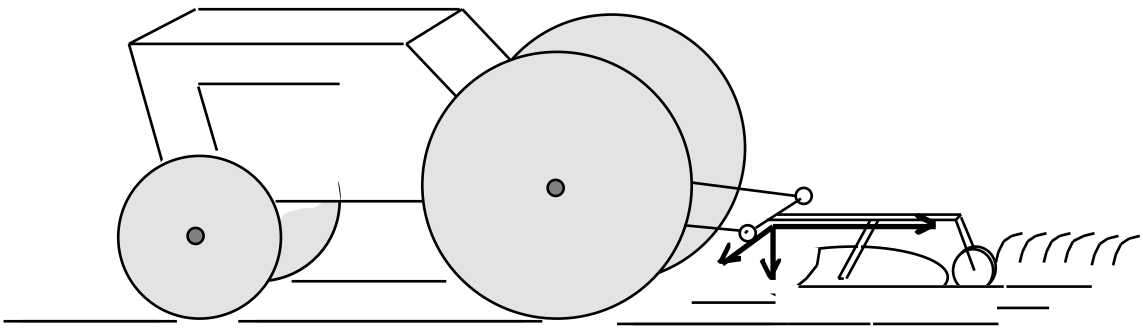


Figure 6.2: Semi-mounted hitch where the front of implement is carried on a horizontal pivot.

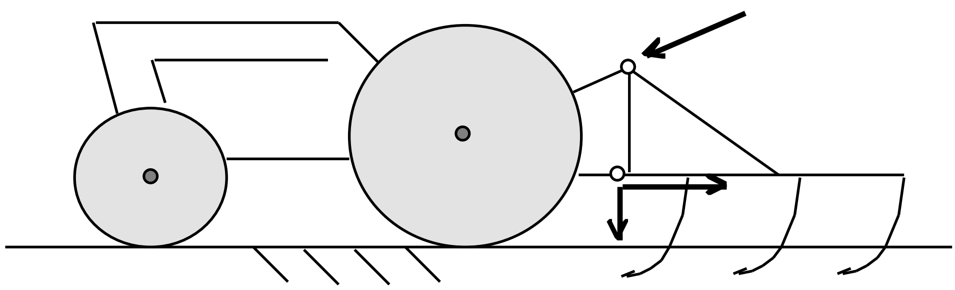


Figure 6.3: Fully mounted, rear three-point linkage hitch.

6.2.2 Hitching systems

(a) Trailed - one point hitch

Here the implement is attached to the tractor at one (drawbar) hitch point. This represents the simplest arrangement, but it provides a minimum in the way of implement control and weight transfer. The implement, which is usually carried on wheels (for support and / or depth control), is free to move in both the horizontal and vertical planes as it follows the varying ground surface.

Two common arrangements can be identified.

(i) where the implement is fully carried on its wheels and its drawbar is pivoted at both ends; the implement force is essentially horizontal, Figure 6.1 (a).

(ii) where the front of the implement (such as in an unbalanced trailer or similar two-wheeled implement) is carried on the tractor drawbar and the rear on a wheel or wheels, Figure 6.1(b). There is usually a significant static vertical component in the implement attachment force and hence the weight transfer from implement to tractor rear wheels is greater than in (i) above.

The trailed hitch is least effective in terms of both weight transfer and implement control when compared with other systems (see Section 6.4.3). The former weakness has been overcome by the development of a weight transfer hitch for trailed implements in which part of the weight of the implement and / or the downward soil forces are supported by the tractor rear wheels. This system is considered in Section 6.4.4(c).

(b) Semi-mounted - two point hitch

In this arrangement the front of the implement is carried on the lower links of the tractor and the rear on a castor wheel as in Figure 6.2.

In the vertical, longitudinal plane the implement is free to pivot about the outer ends of the lower links and hence it behaves as the one point hitch above, ie, it is free to follow ground undulations. It is, however, rigid in the horizontal plane and is therefore frequently used for un-symmetrical implements having side forces, such as mouldboard or disc ploughs, or offset draught forces, such as forage mowers.

There is usually a significant static vertical component in the implement attachment force because part of the weight of the implement and of the downward soil forces are supported by the tractor. Thus weight transfer would be greater than in a corresponding trailed implement; see Section 6.4.3.

(c) Fully mounted - three point hitch

Here the implement is attached to the tractor by means of the three-point linkage as shown in Figure 6.3. In this side view the lower two points are coincident; the upper point is midway between , but above the lower two.

This system totally constrains and allows complete control of the implement. It is not free to swing in space like the trailed implement, nor in the vertical plane like the semi-mounted; it must operate in the position determined for it by the linkage. The exception to this statement is that the implement is usually free to rise, ie, it is not held down by the linkage. If it does rise, it will be due to the upward soil forces being greater than implement weight; it will, however, move in a way determined by the kinematics of the linkage.

In the vertical longitudinal plane (Figure 6.3) the linkage has the form of a mechanism known as a 'four link chain', the characteristics of which are treated in books on kinematics. We can identify the four links as shown in Figure 6.4:

(i) the two lower links (which act as one in the vertical plane)
(ii) the upper or top link
(iii) the implement frame or pedestal
(iv) the tractor chassis.

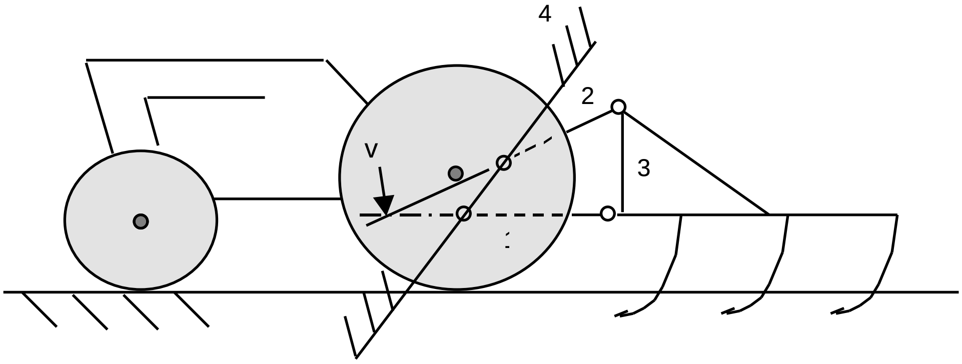


Figure 6.4: Three-point linkage as a four bar 'chain'.

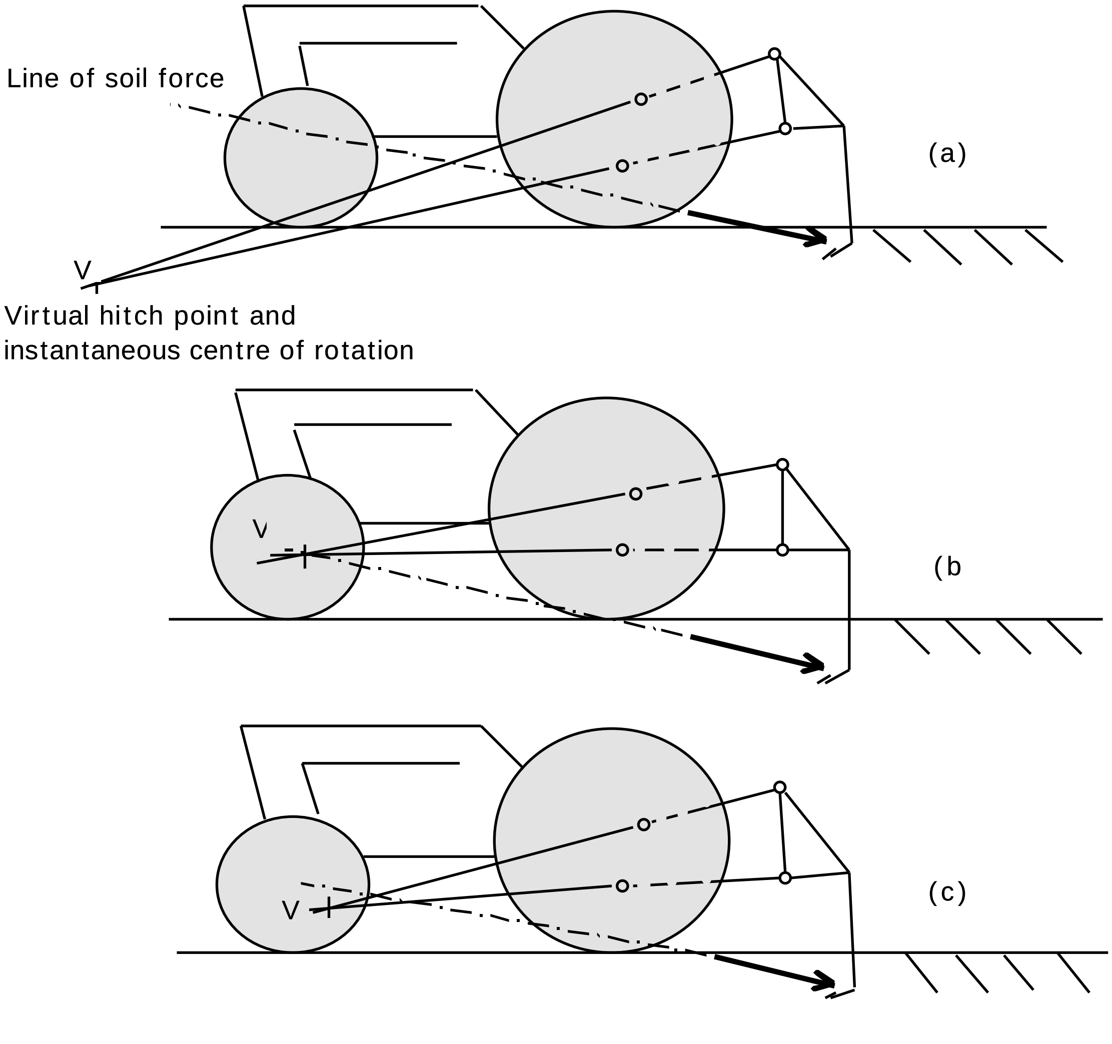


Figure 6.5: Three point linkage as an implement is lowered
(a) near commencement of penetration
(b) in a 'stable' free link condition
(c) restrained above the free link condition

The significant point is 'v' at the intersection of the upper and lower links. When discussing the motion of the implement it is termed the 'instantaneous centre of rotation'; at the instant shown, the implement moves as if it was rotating about that point. The point 'v' itself moves from instant to instant, hence the motion of the implement is quite complex.

When discussing the forces on the implement 'v' is termed the virtual or effective hitch point; at the instant shown, the implement behaves as if it were attached to the tractor at that point.

As an example, Figure 6.5(a) shows a plough on the three-point linkage as it enters the ground. It will be seen that the effective hitch point is below the ground and the line of draft passes above it. The soil force has a clockwise moment about that point, thus the plough is being pulled into the ground. As this occurs, the effective hitch point rises and eventually an equilibrium is reached where the downward force of soil on the plough is just balanced by the upward force of the tractor on the plough. The line of pull passes through the effective hitch point, now above the ground surface, as shown in Figure 6.5(b); this tends to add weight to the rear wheels of the tractor.

The above is termed the 'free link' condition but it is not suitable for normal operation because any variation in the direction of the soil force will cause the implement depth to change . Usually, the linkage is arranged so that the implement reaches the desired working depth before the effective hitch point rises up to the line of draft. The implement is thus kept from reaching the equilibrium condition; the soil forces tend to pull the plough in deeper, but the linkage stops this occurring. The weight of the plough and the downward acting soil forces are thus transferred to the rear wheels of the tractor. The line of draft passes above the effective hitch point, as shown in Figure 6.5(c); the former cannot be located from the latter as in the Figure 6.5(b). Further discussion is given in Dwyer (1974) and Inns (1985).

Problem 6.1

Take measurement of the three-point linkage system on a tractor and associated soil engaging implement. Plot on drawing paper the position of the instantaneous centre of rotation / virtual hitch point if the implement were raised and lowered to below the ground level. Alter the linkage or use another type of implement and repeat the above.

6.3 TRACTOR CHASSIS MECHANICS

The term 'mechanics' here refers to an analysis of the forces that act on the tractor chassis. The major force is that of gravity and is known as the weight. This is sometimes (loosely) given, and spoken of, in units of mass (kg); in engineering analysis (concerned with statics) all such 'weights' should be converted to force units (kN).

6.3.1 Centre of gravity

The centre of gravity is the point at which the whole of the mass and the weight of the tractor may be considered to act. Its location depends on the disposition of the various masses that comprise the tractor. Any analysis of the tractor chassis requires the location of the centre of gravity to be known. It is usually specified in relation to the rear axle as shown by point G in Figure 6.6.

(a) Longitudinal location

The location of the centre of gravity in the longitudinal (x) direction may be found by measuring the weight on the front (W_f) and rear (W_r) wheels.

Application of the force equilibrium condition gives the tractor weight, W:

$$W = W_f + W_r$$

Application of the moment equilibrium condition gives the required longitudinal location, x_r as shown in Figure 6.6(a).

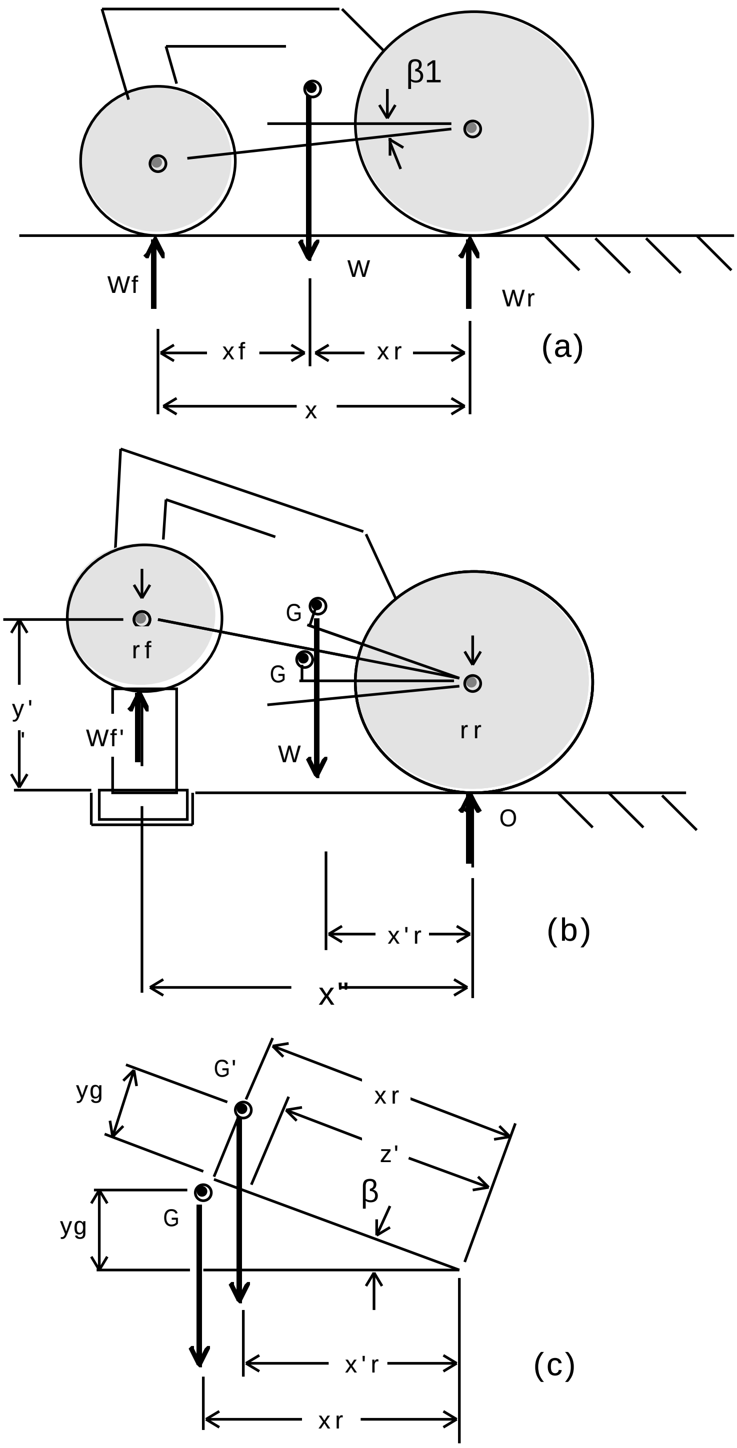


Figure 6.6: Location of centre of gravity of tractor
(a) horizontal location
(b) tractor raised to find vertical location
(c) geometry of position of centre of gravity
Adapted from Barger, et al (1952)

For the tractor take moments about O:

$$W \cdot x_r = W_f \cdot x$$

$$x_r = \frac{W_f}{W} x \qquad (6.1)$$

The wheel base (x) between the front and rear axles is usually given in the manufacturer's specification or can be measured directly.

For most common rear wheel drive tractors x_r is approximately 30 % of x; this is also the % of the static tractor weight that is on the front wheels.

(b) Vertical location

The location of the centre of gravity in the vertical (y) direction is more difficult. The common method is to lift the front (or rear) of the tractor (as shown in Figure 6.6(b)) and measure the weight on the front wheels (W'_f) in the raised condition. The following is similar to Barger et. al., (1952).

Application of the moment equilibrium condition gives the required vertical location, y_g.

For the tractor take moments about O:

$$x'_r = \frac{W'_f}{W} x'' \qquad (6.2)$$

The geometry of the positions of the centre of gravity (Figure 6.1(c)) gives:

$$z = \frac{x'_r}{\cos\beta}$$

$$y_g = \frac{x_r - z}{\tan\beta}$$

Substituting for z gives

$$y_g = \frac{x_r - \dfrac{x'_r}{\cos\beta}}{\tan\beta} \qquad (6.3)$$

where x'_r is as calculated from Equation 6.2 above.

and $\beta = \beta_1 + \beta_2$

$$= \text{atan}\,\frac{r_r - r_f}{x} + \text{atan}\,\frac{y' - r_r}{x''}$$

Inspection of Equation 6.3 shows that if the difference between x_r' and $\dfrac{x_r'}{\cos\beta}$ is to be accurately calculated, β needs to be relatively large and / or accurately determined.

Problem 6.2

By a similar measurement and analysis to the above find the location in the vertical and longitudinal directions of the centre of gravity of a two wheeled tractor or trailer.

6.3.2 Issues in chassis mechanics

Two aspects of the mechanics of the tractor chassis, which are of importance to the performance of the tractor, can be identified:

(a) Weight transfer

For a tractor under dynamic (here meaning 'operating') conditions, the weight on the wheels will, in general, be different from the static values. These changes are termed 'weight transfer' although of course nothing is 'transferred'. The discussion here is limited to the changes in the vertical longitudinal plane, ie, from front to rear and vice versa because these have the greatest influence on tractor performance.

Weight transfer is a normal outcome of the action of the forces generated on the tractor chassis by the ground and by the implement. It occurs whenever and however the tractor is loaded, including the ‘no’ load case where there is some weight transfer due to the torque on the rear wheels required to propel the tractor against the rolling resistance of all the wheels..

It is also normally a desirable outcome because the tractor is designed to take advantage of it by having at least some of the driving wheels at the rear where, for normal forward operation, the increase in rear wheel weight is proportional to the drawbar pull. In reverse gear and in the 'over-run' condition, (the implement pushing the tractor) the forces toward the front of the tractor transfer weight from the rear wheels to the front wheels, a fact which affects the performance of the tractor in this type of work and when braking.

A more detailed discussion of the general subject of weight transfer is given in Gilfillan (1970), Liljedahl et al (1979) and other references given at the end of this Chapter.

(b) Instability

Instability occurs when the weight transfer is sufficient to cause the tractor to tip over rearwards. Impending instability (where the front wheels leave the ground and the tractor is on the point of becoming unstable) is considered here because it is a limiting case of the weight transfer and hence of tractor operation. It is an undesirable situation because it represents loss of steering control and may lead on directly to actual instability. Such a situation is partly avoided by inherent features of the design of the tractor-implement system and partly by its operation in a way that avoids reaching that condition. Usually the wheels slip before instability occurs.

An understanding of the actual process of tipping over in the vertical longitudinal plane which may follow requires a different, more complex dynamic analysis that includes, among other matters, the inertia of the tractor chassis and of the implement, also the inertia and stiffness of the transmission to the rear wheels. This and the analysis of instability in the lateral vertical plane (roll over) are not relevant to tractor performance as such; they are dealt with in Liljedahl et al (1979) and other references given at the end of this Chapter .

6.3.3 Analysis and assumptions

The following analysis of the tractor in the longitudinal, vertical plane is limited to the calculation of wheel weight during steady state operation in normal work (Section 6.4) and to the prediction of the conditions for impending instability (Section 6.5).

Although the tractor and implement are moving, the assumption of steady state operation implies that there are no inertia forces; the forces are doing external work but are not causing any acceleration. Hence the principles of statics and the conditions for static equilibrium of rigid bodies can be applied.

Three independent equations of equilibrium (chosen from the following) can be written:

(i) the sum of the forces in any two perpendicular directions are zero. The two directions usually chosen are those parallel to and perpendicular to the ground surface.

(ii) the sum of the moments about any two points in the vertical longitudinal plane are zero. The two points usually chosen are the wheel / ground contact points or the centres of the wheels.

In simple situations it may be sufficient to consider the whole tractor as a rigid body. Where the external forces are known the weights on the wheels can be calculated directly.

However it is sometimes convenient to consider the tractor as composed of two rigid bodies. One, the drive wheels, rotate about a centre located in the other - the chassis of the tractor. This occurs under the action of the torque acting on them which is internally produced by the engine. Any such analysis must apply appropriate constraints ie, that the forces and moments on each are equal and opposite.

In this analysis and the worked examples, the following simple assumptions are made:

(i) forward motion is uniform; this assumes constant implement forces and no acceleration

(ii) lines of forces on wheels are either tangential or radial or may be resolved as such; wheel sinkage and tyre distortion (but not normal tyre deflection) are neglected

(iii) the tractor is symmetrical about the longitudinal vertical plane; all the forces and moments may be considered to act in this plane

(iv) other forces, such as the change in position of the fuel and oil in the tractor on sloping ground, air resistance and other minor forces are neglected

The analyses of tractors where other more complex assumptions are made are given in the references at the end of this Chapter.

The tractor considered in the general analysis is as shown in Figure 6.7.

The implement force P acts through the point (x', y') at an angle θ to the ground surface. Note that it is not shown 'attached' to the chassis at the rear of the tractor because, in general, it may act on the tractor or attached implement at any point in the plane.

For a trailed hitch shown in Figure 6.1, this point would be the drawbar / implement attachment point. For the tractor in Problem 6.7, P is the weight of a tank and water (a vertical force) carried on the front. Care must therefore be taken to ensure that the direction and the moment of P is correctly included by appropriate choice of θ and the sign for x'.

The solution of the problems given in the following sections will be greatly facilitated by coding of Equations 6.4 and 6.5, etc, on a computer spread sheet.

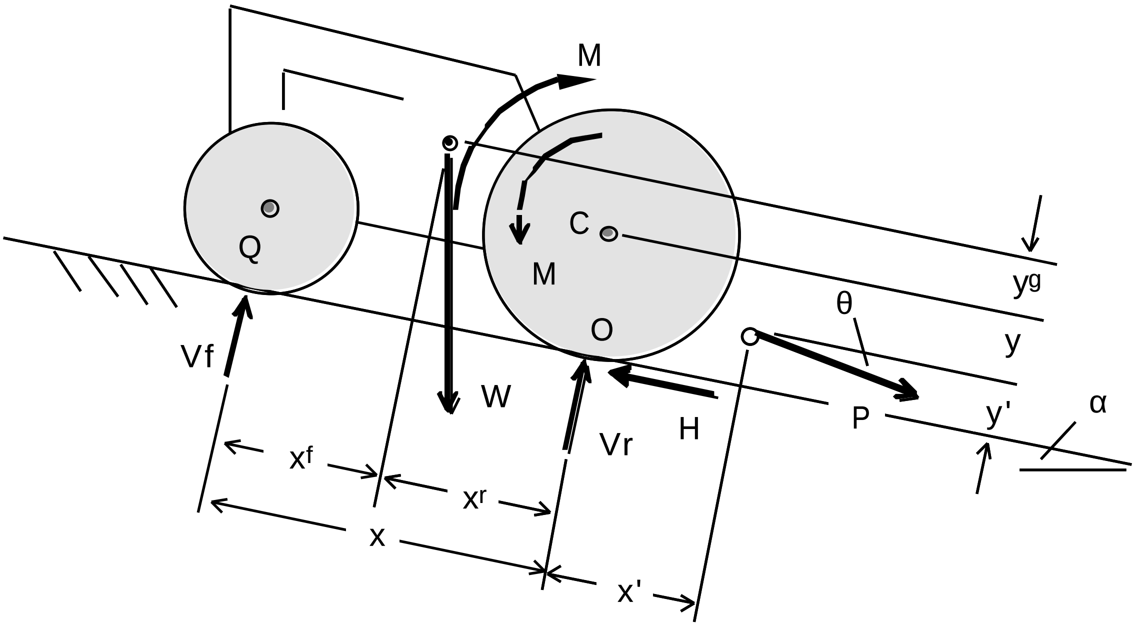


Figure 6.7: Tractor details for weight transfer analysis

6.4 WEIGHT TRANSFER

6.4.1 Four wheel tractor

(a) Analysis

Consider rear wheel drive tractor on a slope as shown in Figure 6.7.

For the tractor[1], take moments about C:

$$V_f\ x + W \sin\alpha\ y_g + P \sin\theta\ x' + M = W \cos\alpha\ x_r + P \cos\theta\ y$$

$$V_f = W \cos\alpha \frac{x_r}{x} + P \cos\theta \frac{y}{x} - \frac{M}{x} - W\sin\alpha \frac{y_g}{x} - P \sin\theta \frac{x'}{x}$$

For the wheels, take moments about C:

$$M = H \cdot r$$

Resolve parallel to the slope:

$$H = W \sin\alpha + P \cos\theta$$

Substitute for M and H above:

$$V_f = W \cos\alpha \frac{x_r}{x} + P \cos\theta \frac{y}{x} - W \sin\alpha \frac{r}{x} - P \cos\theta \frac{r}{x} - W \sin\alpha \frac{y_g}{x} - P \sin\theta \frac{x'}{x}$$

Combining:

$$V_f = W \cos\alpha \frac{x_r}{x} - W \sin\alpha \frac{r+y_g}{x} - P \cos\theta \frac{r-y}{x} - P \sin\theta \frac{x'}{x}$$

$$V_f = W_f - W \sin\alpha \frac{r+y_g}{x} - P \cos\theta \frac{y'}{x} - P \sin\theta \frac{x'}{x} \qquad (6.4)$$

Problem 6.3

Show that the weight on the rear wheels (V_r) perpendicular to the slope is given by:

$$V_r = W_r + W \sin\alpha \frac{r+y_g}{x} + P \cos\theta \frac{y'}{x} + P \sin\theta \frac{x+x'}{x} \qquad (6.5)$$

[1] In the following, the total weight of the tractor (W) and the distance to its centre of gravity (x_r) have been used; this is statically equivalent to using the weight of the body (tractor less rear wheels) and the distance to its centre of gravity.

(b) Explanation of terms

The terms in Equations 6.4 and 6.5 can be identified as follows:

(i) W_f , W_r — the static weight on the wheels when the tractor is on the slope

(ii) $W \sin\alpha \frac{r+y_g}{x}$ — the moment effect of the weight component down the slope, decreasing the front wheel weight and increasing the rear.

(iii) $P \cos\theta \frac{y'}{x}$ — the moment effect of the implement force component down the slope, decreasing the front wheel weight and increasing the rear.

(iv) $P \sin\theta \frac{x'}{x}$ — the moment effect of the implement force component perpendicular to the slope, decreasing the front wheel weight.

(v) $P \sin\theta \frac{x+x'}{x}$ — the direct ($P \sin\theta$) and the moment effect ($P \sin\theta \frac{x'}{x}$) of the implement force component perpendicular to the slope, increasing the rear wheel weight.

Referring to the Equations 6.4 and 6.5, note that the moment effect of the component of the drawbar pull down the slope, $P \cos\theta$, has two effects:

(i) $P \cos\theta \frac{y}{x}$: — increasing V_f and decreases V_r with moment arm y

(ii) $P \cos\theta \frac{r}{x}$ — decreases V_f and increases V_r with moment arm r

The net effect of $P \cos\theta$ is therefore the difference between these two, ie , $P \cos\theta \frac{r-y}{x} = P \cos\theta \frac{y'}{x}$.

This fact gives rise to the idea that if the drawbar pull acts below the rear axle, its moment, $P\cos\theta \cdot y$, increases V_f and holds the front of the tractor down. While this is true, it omits the more important, unrecognised aspect that a usually larger moment, $P\cos\theta \cdot r$, tends to decrease the weight on the front wheels.

Problem 6.4

Check Equations 6.4 and 6.5 by taking moments about the ground contact points O and Q, respectively.

(c) Special cases

The following special cases are of interest:

(i) If y' increases, ie, the point of action (eg, the drawbar) is raised, y decreases and the weight transfer, $P\cos\theta\,\frac{r-y}{x}$ increases; the tractor may reach the condition of impending instability when $V_f = 0$ (Refer Section 6.5)

(ii) If y' = 0, the point of action (the drawbar) is at ground level, y = r; there is no weight transfer due to P.

(iii) If y' is negative, the point of action is below ground level (eg, as is possible with a three point linkage or with the drawbar in a trench), y is greater than r, the term $P\cos\theta\,\frac{y'}{x}$ becomes positive in Equation 6.4 and negative in Equation 6.5, ie, weight is transferred from the rear to the front wheels. (Refer Section 6.4.3.

(iv) If $\theta = 0$, ie, the implement force is parallel to the ground

$$V_f = W\cos\alpha\,\frac{x_r}{x} - W\sin\alpha\,\frac{r+y_g}{x} - P\,\frac{y'}{x} \qquad V_r = W\cos\alpha\,\frac{x_f}{x} + W\sin\alpha\,\frac{r+y_g}{x} + P\,\frac{y'}{x}$$

(v) If also, $\alpha = 0$, ie, the ground is horizontal

$$V_f = W\,\frac{x_r}{x} - \frac{Py'}{x} = W_f - \frac{Py'}{x} \qquad V_r = W\,\frac{x_f}{x} + \frac{Py'}{x} = W_r + \frac{Py'}{x}$$

(vi) If also, P = 0, ie, there is no implement force

$$V_f = W\,\frac{x_r}{x} = W_f \qquad V_r = W\,\frac{x_f}{x} = W_r$$

Problem 6.5

Repeat the analysis in Section 6.4.1 for the tractor travelling down the slope where the implement force acts forwards and downwards (as when towing an unbalanced trailer); show that the wheel weights are:

$$V_f = W_f + W\sin\alpha\,\frac{r+y_g}{x} + P\cos\theta\,\frac{y'}{x} - P\sin\theta\,\frac{x'}{x} \qquad (6.6)$$

$$V_r = W_r - W\sin\alpha\,\frac{r+y_g}{x} - P\cos\theta\,\frac{y'}{x} + P\sin\theta\,\frac{x'+x}{x} \qquad (6.7)$$

Problem 6.6

Consider the Farmland tractor with a spray tank mounted on the three-point linkage at the rear.

The following data apply:

Weight of spray tank when empty	= 60 kg
Centre of gravity of the tank and water	= 1.5 m from the rear axle
	= 1.0 m from the ground

(i) If there is 210 kg of water in the tank, what is the weight on the front wheels for the unit moving on horizontal ground?

(ii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving up a 10^o slope and the weight on the front wheels is to not be less than 4kN?

(iii) What will be the maximum weight on the front wheels and the tractive coefficient as the tractor empties the spray tank while travelling down a 10^o slope ?

Solution Part (ii)

From Equation 6.4:

$$V_f = W_f - W\sin\alpha\,\frac{r+y_g}{x} - P\cos\theta\,\frac{y'}{x} - P\sin\theta\,\frac{x'}{x}$$

$$P = \frac{W\cos\alpha\, x_r - W\sin\alpha\,(r+y_g) - V_f\, x}{\cos\theta\ y' + \sin\theta\ x'}$$

$$= \frac{27.9\,(.532 - .133) - 7.52}{.174 + 1.48} = 2.18 \text{ kN} = 224 \text{ kg}$$

Weight of water = 224 - 60 = 164 kg

From Equation 6.5

$$V_r = W_r + W\sin\alpha\,\frac{r+y_g}{x} + P\cos\theta\,\frac{y'}{x} + P\sin\theta\,\frac{x+x'}{x}$$

$$= 27.9(.985\,\frac{1.34}{1.88} + .174\,\frac{.765}{1.88}) + 2.18\,(.174\,\frac{1}{1.88} + .985\,\frac{3.38}{1.88})$$

$$= 19.6 + 1.97 + .20 + 3.89$$

$$= 25.6 \text{ kN}$$

$$\psi' = \frac{W\sin\alpha + P\cos\theta}{Vr} = \frac{27.9x\ .174 + 2.18\ x\ .174}{25.6} = 0.20$$

Answers: (i) 5.92 kN; (ii) 164 kg, 0.20; (iii) 9.48kN, -0.27

Problem 6.7

Repeat Problem 6.6 with the spray tank mounted on the front of the tractor with its centre of gravity 1.5 m from the front axle.

(i) If there is 210 kg of water, what is the weight on the front wheels for the unit moving on horizontal ground?

(ii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving up a 10^o slope and the front wheel weight is to not exceed 10 kN?

(iii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving down a 10^o slope and the front wheel weight is to not exceed 14 kN?

Answers: (i) 12.8 kN; (ii)187 kg, 0.26; (b) 165 kg; -0.33

Problem 6.8

Consider the Farmland tractor operating up a slope $\alpha = 15^o$ with a drawbar pull angle $\theta = 10^o$.

Use Equation 6.5 to calculate the :
(i) maximum drawbar pull if the tractive coefficient ψ (based on the total tractive force) = 0.8
(ii) rear wheel weight
(iii) percentage contributions of the terms in Equation 6.5 to the tractive force.

Note: An iterative method is required to solve this problem because the rear wheel weight depends on the drawbar pull (due to weight transfer) and the drawbar pull (as determined by the tractive coefficient) depends on the rear wheel weight. Assume an initial value for P and calculate V_r, H and then P; if the initial value of P is carefully chosen, the answer will be obtained with sufficient accuracy with two iterations.

Answers: (i) 17.1 kN; (ii) 30.1 kN; (iii) 64%, 10%, 13%, 13%

6.4.2 Weight transfer with rolling resistance

The above analysis neglects any effect of rolling resistance. We may, however, include this by introducing a force acting along the slope (opposite the direction of motion) as a further force to be overcome by the tractor.

As discussed in Section 4.3.3 the rolling resistance may be expressed in terms of a coefficient (ρ) as

Rolling resistance = ρ . Weight on wheel

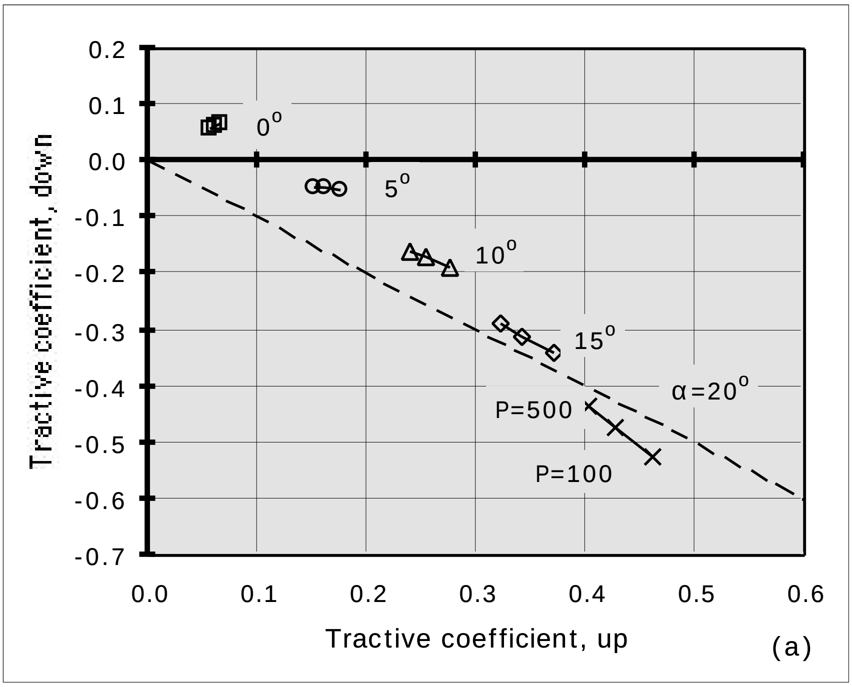
Here the weight will be the wheel weights perpendicular to the slope, ie, V_f and V_r as given by Equations 6.4 and 6.5 above. The rolling resistance for the tractor may be estimated by combining the effect on the front and rear wheels by considering a coefficient for the tractor as a whole.

$$R = \rho (V_f + V_r)$$

$$= \rho (W \cos\alpha + P \sin\theta)$$

The total tractive force

$$H = W \sin\alpha + P \cos\theta + \rho (W \cos\alpha + P \sin\theta)$$



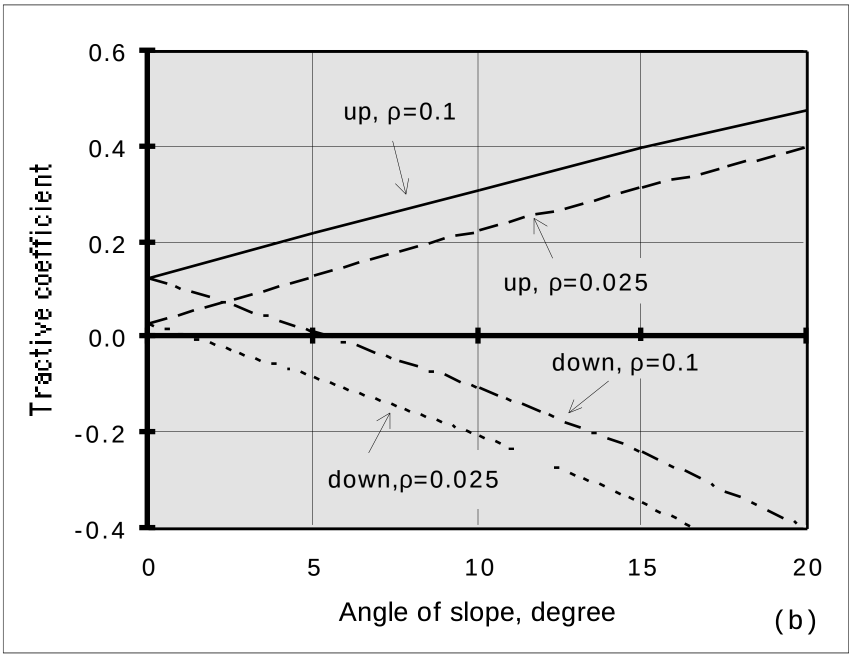


Figure 6.8: Tractive coefficients required for the Farmland tractor working up and down the slope :
(a) carrying a weight of 100, 300 and 500kg with rolling resistance coefficient of 0.05
(b) Carrying a weight of 300kg with rolling resistance coefficient of 0.025 (bitumen road)and 0.1 (ploughed soil).

We can specify the tractive force required (for a rear wheel drive tractor) in terms of the gross tractive coefficient.

$$\psi' = \frac{\text{Tractive force}}{\text{Rear wheel weight}}$$

$$= \frac{W \sin\alpha + P\cos\theta + \rho\,(W\cos\alpha + P\sin\theta)}{W_r + W\sin\alpha\,\frac{r+y_g}{x} + P\cos\theta\,\frac{y'}{x} + P\sin\theta\,\frac{x+x'}{x}} \qquad (6.8)$$

Problem 6.9

The Farmland tractor carries a fertilizer distributor mounted on the rear three-point linkage.
The following data apply:

Centre of gravity of distributor and fertilizer, m:	1.5 behind tractor rear axle 1.0 above ground
Total weight of the distributor and fertilizer, kg:	100 (empty), 300 500 (full)
Rolling resistance coefficient	0.025 (bitumen road), 0.050 (firm surface) 0.1 (ploughed soil)
Angle of slope (up and down), ^{o}	0, 5, 10, 15, and 20

Calculate the traction coefficient required to drive the tractor and distributor under various conditions. Hence identify conditions where it may be possible and safe to drive up a slope but unsafe to drive down it.

Solution

Results for some conditions which are given in Figure 6.8(a) for r=0.05 (firm conditions) show that the tractive coefficient required:
(i) increases with the angle of slope
(ii) decreases with weight carried, particularly for larger angles

Figure 6.8(b) shows that the tractive coefficient depends on the angle of slope and the rolling resistance. In the example given for load =300 kg and ρ = 0.025 (bitumen road), ψ'(down) > ψ' (up) for slope >12^o.

Problem 6.10

Repeat Problem 6.9 with the distributor mounted on the front of the tractor .

Assume that the centre of gravity of distributor and fertilizer is 1.5 m in front of the front axle and 1.0 m above the front wheel ground contact point .

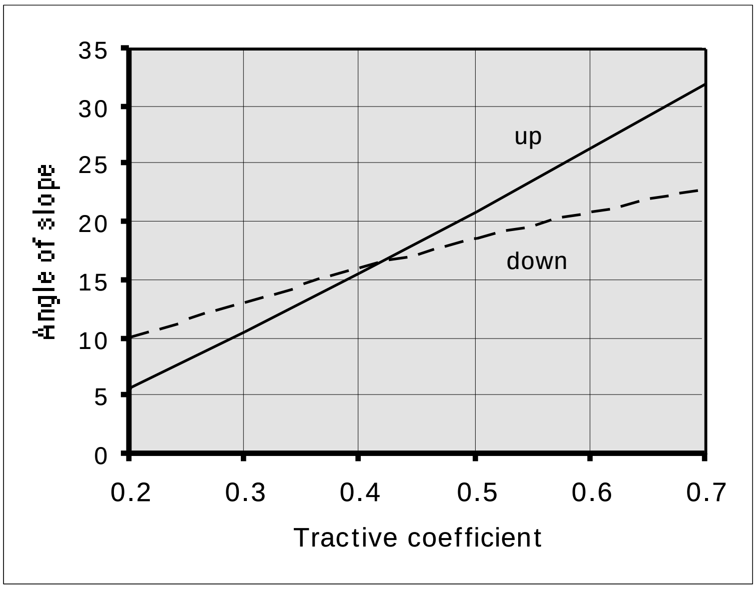


Figure 6.9: Slopes that can be negotiated for various traction coefficients, Problem 6.11

Problem 6.11

The Farmland tractor operates with zero drawbar pull on a slope α . The maximum gross tractive coefficient is ψ' and the coefficient of rolling resistance for the tractor as a whole is ρ.

(i) What is the maximum slope that the tractor can travel up without exceeding the maximum tractive force.

Resolving along the slope, Figure 6.7:

$$H = W \sin\alpha + W \cos\alpha \, \rho$$

At maximum gross tractive coefficient :

$$H = V_r \, . \, \psi'$$

The dynamic weight V_r on the rear wheels in operation is given by moments about Q:

$$V_r \, . \, x = W \cos\alpha \, x_f + W \sin\alpha \, (r + y_g) \,]$$

$$V_r = \frac{W \cos\alpha \, x_f + W \sin\alpha \, (r + y_g)}{x}$$

Substitute for H and V_r above:

$$W \sin\alpha + W \cos\alpha \, \rho = [\frac{W \cos\alpha \, x_f + W \sin\alpha \, (r + y_g)}{x}] \, \psi'$$

$$W \sin\alpha \, [1 - \frac{(r + y_g)}{x} \psi'] = W \cos\alpha \, [\frac{x_f}{x} \psi' - \rho]$$

$$\tan\alpha_u = \frac{\psi' \, x_f - \rho \, x}{x - \psi'(r+y_g)} \qquad (6.9)$$

(ii) Show that the maximum slope that the tractor can travel <u>down</u> without exceeding the maximum tractive force is:

$$\tan\alpha_d = \frac{\psi' \, x_f + \rho \, x}{x + \psi' \, (r+y_g)} \qquad (6.10)$$

(iii) Plot tan α_u and tan α_d for values of ψ' between 0.2 and 0.7 and $\rho = 0.05$ and discuss the meaning of these results.

Answer (iii) See Figure 6.9

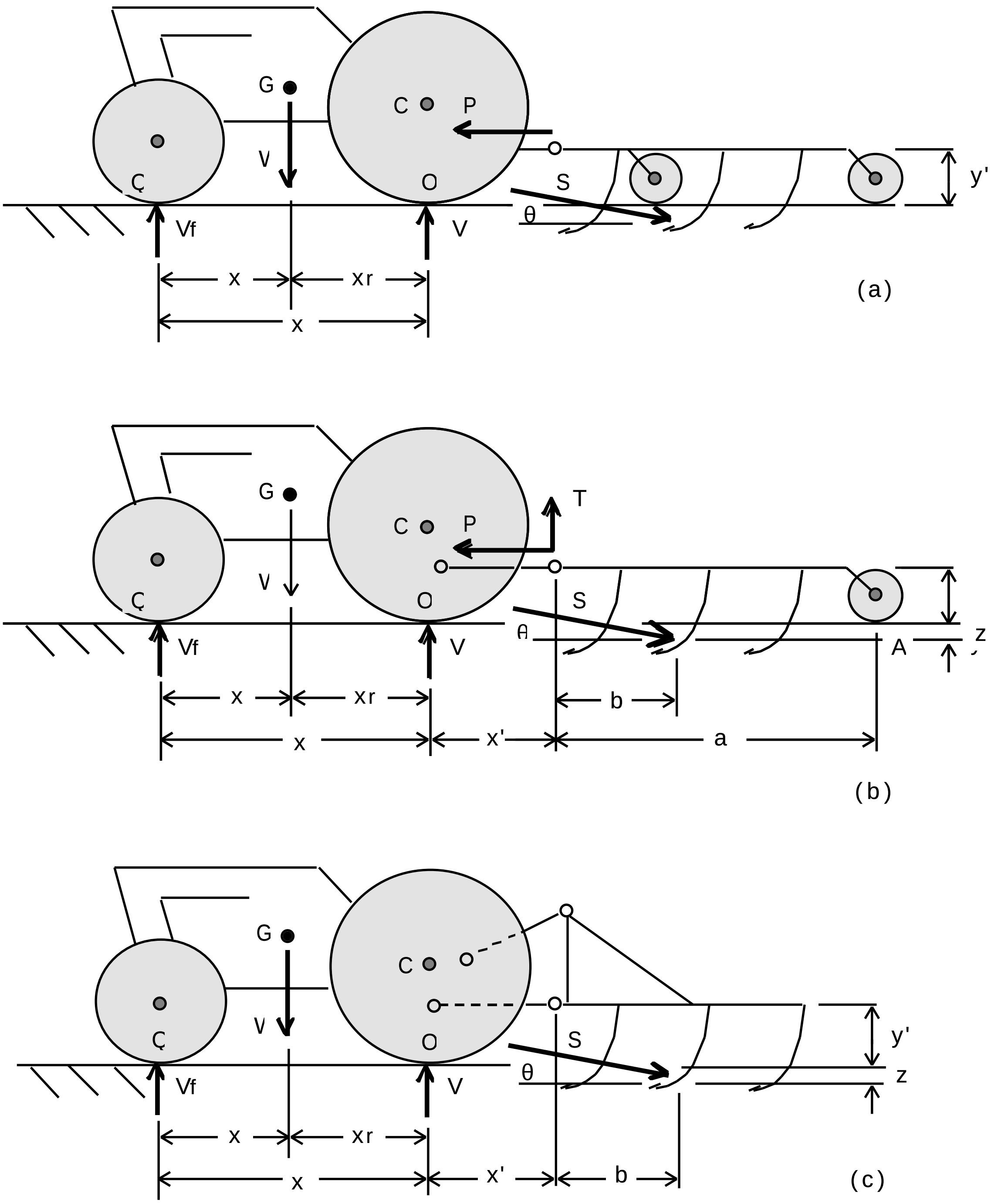


Figure 6.10: Weight transfer with various hitching systems;
(a) trailed; (b) semi-mounted; (c) fully mounted

6.4.3 Weight transfer with hitching systems

(a) Analysis

Considering the three common hitching systems described in Section 6.2.2 above, we are now in a position to evaluate them with respect to weight transfer, ie, the increase in the weight on the rear wheels as a result of the implement forces. This analysis does not take into account the weight of the implement, which is more significant for the mounted and semi-mounted systems than for the trailed. However, it provides a valid comparison of the relative advantages of weight transfer of the three systems on the basis of the soil forces and of the conditions under which these advantages will be achieved.

Consider an identical cultivator, as shown in Figure 6.10 hitched, in the following ways:
(i) trailed on its own wheels
(ii) semi-mounted on the lower links of the tractor and a rear wheel
(ii) fully mounted on the three-point linkage.

In order to compare them it is necessary to determine the dynamic weight on the front and rear wheels of the tractor for each system; the same soil force S, acting at an angle θ to the ground surface as shown, is assumed for each .

(i) Trailed

Resolving horizontally:

$$P = S\cos\theta$$

Moments about Q for the tractor:

$$V_r\, x = W\, x_f + P\, y'$$

$$V_r = W_r + \frac{S\cos\theta\; y'}{x} \qquad (6.11)$$

And

$$V_f = W_f - \frac{S\cos\theta\; y'}{x} \qquad (6.12)$$

Weight transfer will occur if $V_r > W_r$ ie, if y' is positive, ie, if the drawbar is above ground level; it will be increased by increasing the drawbar height, y'.

For a consideration of the implications of this, see the more general analysis of impending instability given in Section 6.5.

(ii) Semi-mounted

Resolving horizontally:

$$P = S\cos\theta$$

The dynamic weight T on the tractor drawbar is given by moments about A for the cultivator:

$$T\, a = S\sin\theta\,(a-b) + P\, y' + S\cos\theta\, z$$

where b gives the horizontal location of the soil force.

Substituting for P

$$T = S\sin\theta\,\frac{a-b}{a} + S\cos\theta\,\frac{z+y'}{a}$$

The dynamic weight on the rear wheels is given by moments about Q:

$$V_r\, x = W\, x_f + P\, y' + T\,(x+x')$$

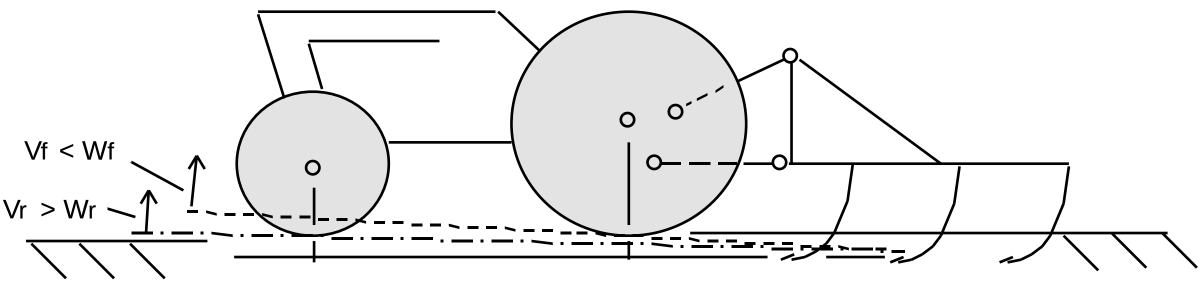


Figure 6.11 Conditions for weight transfer with fully mounted implement.

HITCHING SYSTEM	CONDITION FOR WEIGHT TRANSFER	EXPLANATION
TRAILED	$V_r > W_r$ unless y' negative	Drawbar above ground level
SEMI MOUNTED	$V_r > W_r$ always	- -
MOUNTED	$V_r > W_r$ if $\tan\theta > \frac{z}{x+x'+b}$ $V_f < W_f$ if $\tan\theta > \frac{z}{x'+b}$	Line of soil force passes above:- front wheel/ ground contact point rear wheel / ground contact point

Table 6.1: Summary of conditions for weight transfer with various hitching systems

Substituting for T and P:

$$V_r\ x = W\,x_f + S\cos\theta\ y' + S\sin\theta\,\frac{(a-b)(x+x')}{a} + S\cos\theta\,\frac{(z+y')(x+x')}{a}$$

$$V_r = W_r + S\sin\theta\,\frac{(a-b)(x+x')}{ax} + S\cos\theta\left[\frac{y'}{x} + \frac{(z+y')(x+x')}{ax}\right]$$

$$= W_r + S\sin\theta\,\frac{(a-b)(x+x')}{ax} + S\cos\theta\,\frac{ay' + (z+y')(x+x')}{ax} \qquad (6.13)$$

Problem 6.12

Show that the weight on the front wheels of the tractor with semi-mounted implement is given by:

$$V_f = W_f - S\sin\theta\,\frac{(a-b)\,x'}{ax} - S\cos\theta\,\frac{ay' + (z+y')x'}{ax} \qquad (6.14)$$

Weight transfer will occur if $V_r > W_r$ which will always occur unless one of the following terms is negative and greater in magnitude than the other.

The first term will be negative if b > a, ie, the soil force is behind the wheel. The second will be negative if y' is negative (below ground level) and greater than z or z is negative (above ground level) and greater than y'.

All of these conditions are unlikely to occur for a semi-mounted implement, hence weight transfer will always occur.

(iii) Mounted

The dynamic weight V_r on the rear wheels is given by moments about Q for the tractor / implement system as a whole:

$$V_r\ x + S\cos\theta\ z = W\,x_f + S\sin\theta\,(x+x'+b)$$

$$V_r = W_r + S\sin\theta\,\frac{x+x'+b}{x} - S\cos\theta\,\frac{z}{x} \qquad (6.15)$$

The dynamic weight V_f on the front wheels is given by moments about O for the tractor / implement system as a whole:

$$W\,x_r + S\cos\theta\ z = V_f\ x + S\sin\theta\,(x'+b)$$

$$V_f = W_f - S\sin\theta\,\frac{x'+b}{x} + S\cos\theta\,\frac{z}{x} \qquad (6.16)$$

Increasing the length of mounted implements (hence increasing b) will increase the weight transfer to the rear wheels due to the direct effect (S sin θ) and the moment effect ($S\sin\theta\ \frac{x'+b}{x}$) from the front wheels. The limit will be the length and weight that will still allow the tractor to lift the implement without itself tipping up; weights may be added to the front of the tractor to avoid this.

Weight transfer will occur if:

$$V_r > W_r$$

$$S \sin\theta \frac{x+x'+b}{x} > S \cos\theta \frac{z}{x}$$

$$\tan\theta > \frac{z}{x+x'+b} \qquad (6.17)$$

This implies that weight transfer to the rear wheels will occur if the soil force passes above the front wheel / ground contact point, Figure 6.11.

The above includes the contribution of the vertical component of the soil force (S sinθ) to the rear wheel weight.

Another measure associated with weight transfer from the front wheels in the mounted system is the condition that

$$V_f < W_f$$

$$S \sin\theta \frac{x'+b}{x} > S \cos\theta \frac{z}{x}$$

$$\tan\theta > \frac{z}{x'+b} \qquad (6.18)$$

This implies that weight transfer from the front wheels to the rear will occur if the soil force passes above the rear wheel / ground contact point. Further, weight transfer will increase as b increases, ie, the implement gets longer.

(iv) Summary

A summary of the results of this analysis is given in Table 6.1.

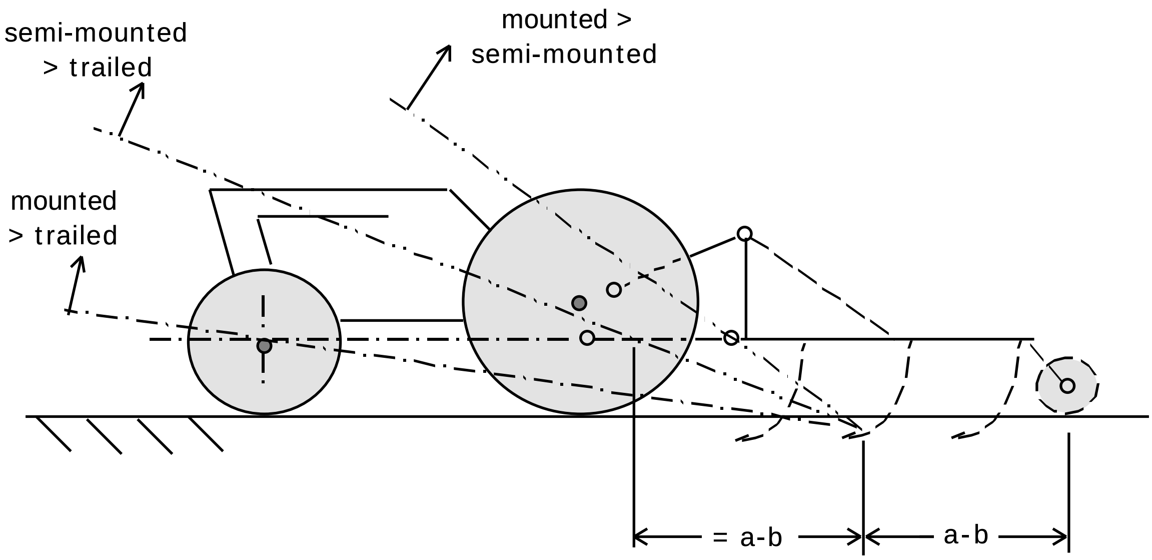


Figure 6.12: Comparison of hitching systems on the basis of weight transfer

TOP GREATER THAN SIDE if line of soil force passes above:	**MOUNTED**	
SEMI MOUNTED	Axis of lower hitch points: $\tan\theta > \frac{z+y'}{b}$	**SEMI MOUNTED**
TRAILED	Intersection, drawbar line & vertical line through front axle: $\tan\theta > \frac{z+y'}{x+x'+b}$	Intersection, drawbar line & vertical line as far in front of soil force as wheel is behind it: $\tan\theta > \frac{z+y'}{a-b}$

Table 6.2 Summary comparison of weight transfer effects for different hitching systems

(b) Comparison of hitching systems

We seek to determine the conditions under which the weight transfer for each system in Section 6.4.3 (a) is greater than the one above it.

(i) Condition for V_r (mounted) greater than V_r (semi-mounted) :

$$S \sin\theta \frac{x+x'+b}{x} - S \cos\theta \frac{z}{x} > S \sin\theta \frac{(a-b)(x+x')}{ax} + S \cos\theta \left[\frac{ay' + (z+y')(x+x')}{ax} \right]$$

$$\sin\theta \left[\frac{x+x'+b}{x} - \frac{(a-b)(x+x')}{ax}\right] > \cos\theta \left[\frac{z}{x} + \frac{ay' + (z+y')(x+x')}{ax}\right]$$

$$\sin\theta\,[b(a+x+x')] > \cos\theta\,[(z+y')(a+x+x')]$$

$$\tan\theta > \frac{(z+y')}{b} \qquad (6.19)$$

For the weight transfer of the mounted implement to be greater than that for the semi-mounted, the soil force must pass above the lower hitch points; see Figure 6.12.

(ii) Condition for V_r (mounted) greater than V_r (trailed) :

$$S \sin\theta \frac{x+x'+b}{x} - S \cos\theta \frac{z}{x} > S \cos\theta \frac{y'}{x}$$

$$\sin\theta \frac{x+x'+b}{x} > \cos\theta \frac{z+y'}{x}$$

$$\tan\theta > \frac{z+y'}{x+x'+b} \qquad (6.20)$$

For the weight transfer for the mounted implement to be greater than that for the trailed, the soil force must pass above the intersection of the drawbar line and a vertical line through the front axle; see Figure 6.12.

(iii) Condition for V_r (semi-mounted) greater than V_r (trailed) :

$$S \sin\theta \frac{(a-b)(x+x')}{ax} + S \cos\theta \left[\frac{ay' + (z+y')(x+x')}{ax}\right] > S \cos\theta \frac{y'}{x}$$

$$\sin\theta \frac{(a-b)(x+x')}{ax} > \cos\theta \left[\frac{y'}{x} - \frac{ay' + (z+y')(x+x')}{ax}\right]$$

$$\tan\theta > \frac{z+y'}{a-b} \qquad (6.21)$$

For the weight transfer for the semi-mounted cultivator to be greater than that for the trailed, the soil force must pass above the intersection of the drawbar line and vertical line through a point as far forward of the soil force as the wheel of the semi-mounted cultivator is behind it; see Figure 6.12.

(iv) Summary

A summary of the results of this analysis is given in Table 6.2

The above conditions are likely to be met with implements which have;

(i) a soil force with significant vertical component, such as mouldboard ploughs, compared to those with a more horizontal force, such as cultivators.

(ii) long implements for which b is large.

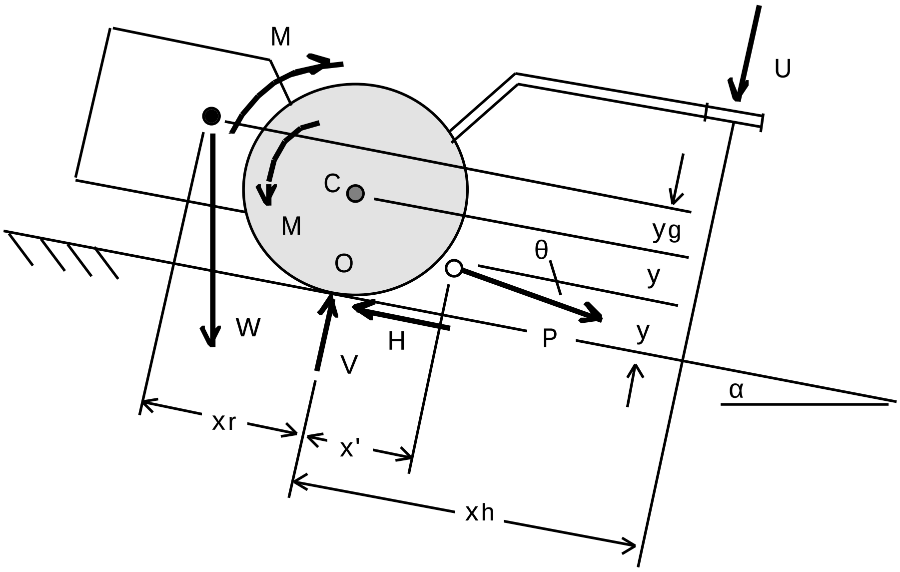


Figure 6.13 Two wheeled tractor dimensions relevant to weight transfer analysis

6.4.4 Other examples

(a) Two wheel (walking) tractor

The wheels of a two wheel (or so-called 'walking') tractor are usually driven through a V belt and / or chain drive as shown in Figure 1.3. The mechanics of its chassis are the same, in principle, as the conventional four-wheel tractor but here the tractor chassis requires 'support' when pulling a drawbar load. This will usually be provided by one or more of the following:

(i) a tool, implement or trailer at the rear
(ii) a wheel at the rear
(iii) a counter balance weight at the front
(iv) the operator through the handles

Consider the two-wheel tractor as shown in Figure 6.13 on a slope with an angled pull through the drawbar. Normally the location of the centre of gravity would be such that with no drawbar pull the tractor would tip forwards and a counteracting force U acting down on the handles would be required. When a drawbar pull acts the net moment on the chassis will be clockwise as in Figure 6.13 and so the tractor tends to balance itself.

(i) With zero drawbar pull:

For the tractor, take moments about O:

$$U_o \; x_h + W \sin\alpha \,(r + y_g) = W \cos\alpha \; x_r$$

$$U_o = W \cos\alpha \frac{x_r}{x_h} - W \sin\alpha \frac{r + y_g}{x_h} \qquad (6.22)$$

This force must act downward as shown if the centre of gravity of the tractor, counter weight and implement are forward of the axle.

(ii) With drawbar pull:

For the tractor resolve parallel to the slope:

$$H = P \cos\theta + W \sin\alpha$$

Take moments about C for the wheels:

$$M = H \, . \, r$$

Moments about C for the tractor:

$$M + W \sin\alpha \; y_g + P \sin\theta \; x' + U \; x_h = W \cos\alpha \; x_r + P \cos\theta \,(r - y')$$

Substitute for H and M from above:

$$P \cos\theta \; r + W \sin\alpha \; r + W \sin\alpha \; y_g + P \sin\theta \; x' + U \, x_h = W \cos\alpha \; x_r + P \cos\theta \,(r - y')$$

$$U = W \cos\alpha \frac{x_r}{x_h} - W \sin\alpha \frac{r + y_g}{x_h} - P \cos\theta \frac{y'}{x_h} - P \sin\theta \frac{x'}{x_h} \qquad (6.23)$$

$$= U_o - P \cos\theta \frac{y'}{x_h} - P \sin\theta \frac{x'}{x_h} \qquad (6.24)$$

Problem 6.13

For the two-wheel tractor on slope and with angled drawbar pull, show that the normal wheel weight is:

$$V = W\cos\alpha \frac{x_h + x_r}{x_h} - W\sin\alpha \frac{r + y_g}{x_h} - P\cos\theta \frac{y'}{x_h} + P\sin\theta \frac{x_h - x'}{x_h} \qquad (6.25)$$

For the convenient operation of such a tractor it would be desirable to arrange that the force U = 0 under operating conditions. Examination of Equation 6.23 (for simplicity with $\alpha = 0$) shows that this will depend on balancing the moment of the weight and of the drawbar pull .

$$W\cos\alpha \; x_r = P\cos\theta \; y' + P\sin\theta \; x'$$

To achieve this it is common to attach a large weight at the front of the tractor, the position of which is adjustable with respect to the axle (equivalent to changing x_r) to achieve the desired balance.

$$x_r = \frac{P\cos\theta \; y' + P\sin\theta \; x'}{W\cos\alpha}$$

Problem 6.14

Show that for the walking tractor with $\alpha = 0$ and $\theta = 0$, the condition for U=0 at maximum drawbar pull is that $x_r = \psi' y'$.

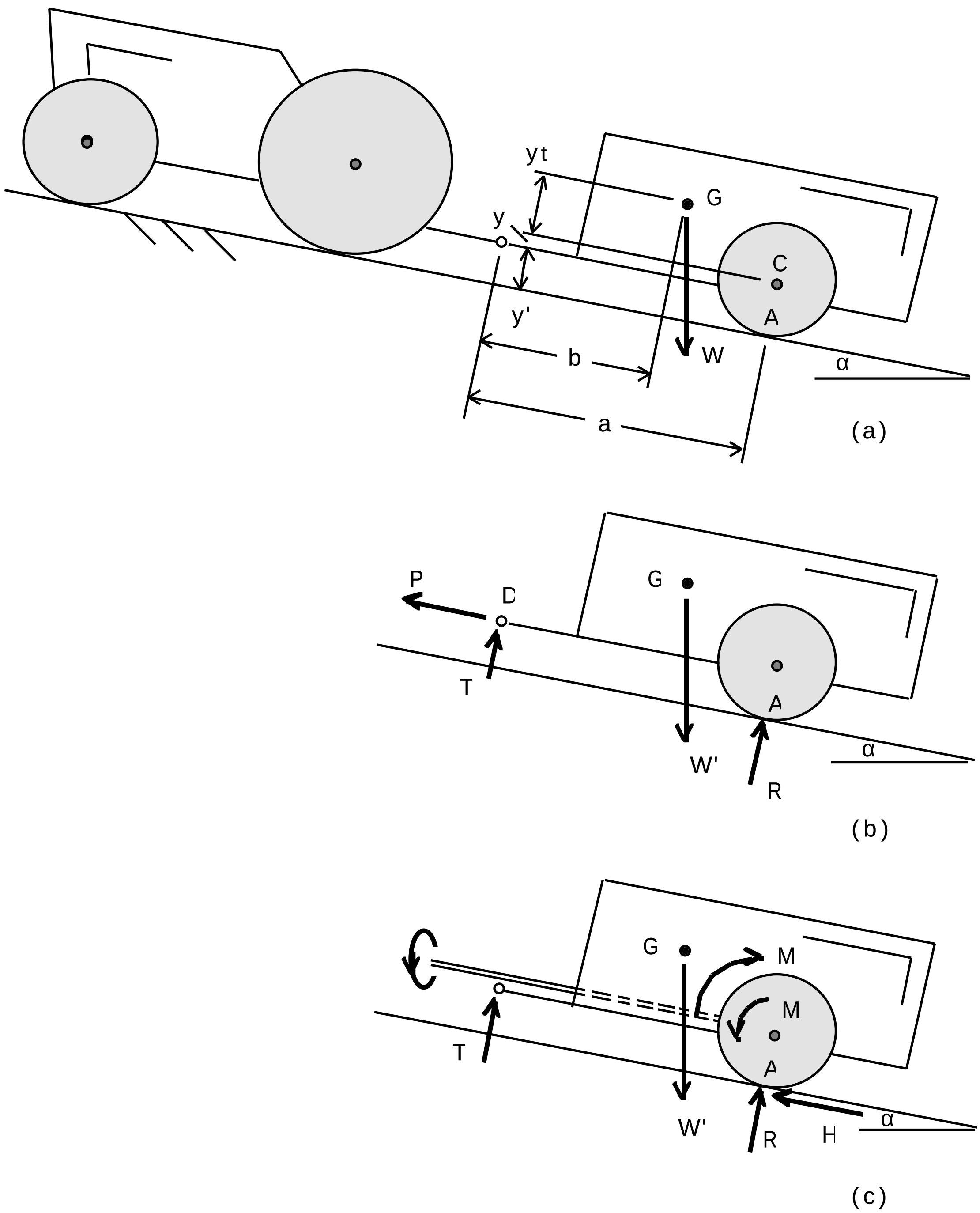


Figure 6.14: Details of PTO driven trailer for analysis

(b) PTO driven trailer

The PTO driven trailer as shown in Figure 6.14 (a) can be pulled up a slope by a tractor (6.14 (b)) or the wheels can also be driven via a drive shaft from the PTO, (6.14(c)).

(i) Pulled

Consider the trailer being pulled up the slope as in Figure 6.14(b).

Resolve along the slope for the trailer:

$$P = W' \sin\alpha$$

Moments about D for the trailer:

$$R \,.\, a = W' \cos\alpha \; b + W' \sin\alpha \;(y + y_t)$$

$$R = W' \cos\alpha \frac{b}{a} - W' \sin\alpha \frac{y+y_t}{a}$$

Resolve perpendicular to the slope for the trailer:

$$R + T = W' \cos\alpha$$

Substitute for R:

$$T = W' \cos\alpha - W' \cos\alpha \frac{b}{a} + W' \sin\alpha \frac{y+y_t}{a}$$

$$= W' \cos\alpha \frac{a-b}{a} + W' \sin\alpha \frac{y+y_t}{a}$$

(ii) Driven

Consider now the wheels being driven so that the drawbar pull on the tractor is brought to zero as in Figure 6.14(c). Determine the tractive coefficient required for the trailer wheels.

Moments about C for the trailer:

$$W' \sin\alpha \; y_t + T \,.\, a + M = W' \cos\alpha \;(a-b)$$

Moments about C for the trailer wheels:

$$M = H \,.\, r'$$

Resolve along the slope:

$$H = W' \sin\alpha$$

Resolve perpendicular to the slope for the trailer;

$$R + T = W' \cos\alpha$$

Substitute for T and M above

$$W' \sin\alpha \; y_t + (W' \cos\alpha - R) \; a + W' \sin\alpha \; r = W' \cos\alpha \;(a-b)$$

$$R = W' \sin\alpha \frac{r + y_t}{a} + W' \cos\alpha \frac{b}{a}$$

$$\psi' = \frac{H}{R} = \frac{W' \sin\alpha}{W' \sin\alpha \frac{r + y_t}{a} + W' \cos\alpha \frac{b}{a}} = \frac{a \tan\alpha}{(r + y_t) \tan\alpha + b} \qquad (6.26)$$

The required tractive coefficient thus depends in a complex way on the slope angle α and the position of the wheels and the centre of gravity.

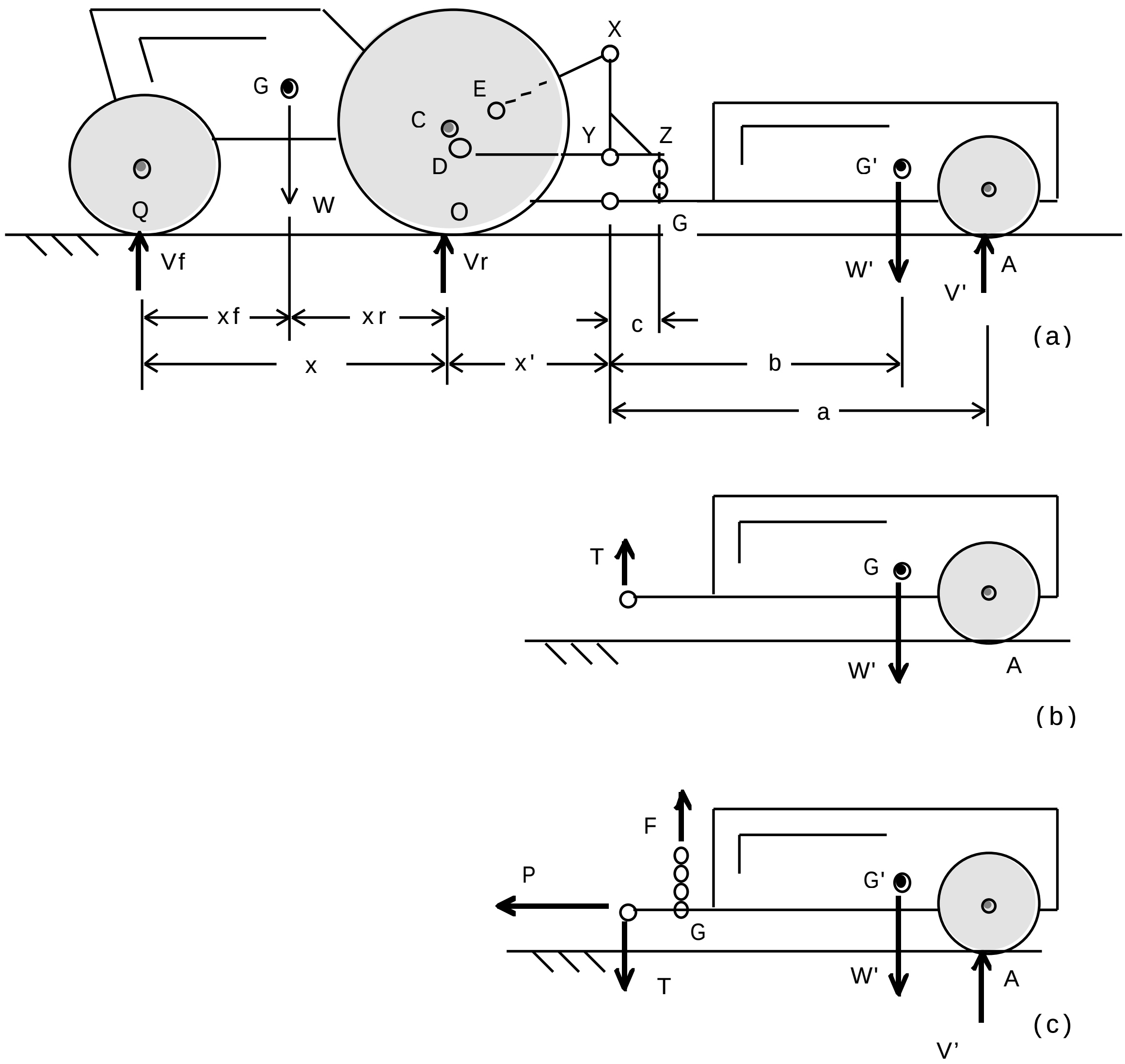


Figure 6.15: Trailed weight transfer hitch; (a) and (b) without lift; (c) with lift

(c) Trailed implement weight transfer system

It was shown above that the trailed hitch is least effective in terms of weight transfer. This deficiency has been overcome by the development of a weight transfer hitch, in which part of the weight of trailed implements, and / or downward soil forces, are supported by the tractor rear wheels (Persson, 1967; Hockey,1961-62).

The principle of one common system is illustrated in Figure 6.15. The rigid link XYZ is attached to the three-point linkage DY and EY; Z is connected to the implement drawbar with a flexible link ZG. In operation, the three-point linkage applies a lifting force F to the implement; this is set by the operator and is kept constant by a hydraulic valve even when the tractor pitches with respect to the implement. This support (but not lifting movement) of the implement transfers some implement weight, as well as some of the tractor front wheel weight, onto the rear wheels.

Assume the weight transfer hitch is attached to an unbalanced trailer as shown in Figure 6.15.

It is required to determine the weight on the rear wheels of the tractor when there is a force F in the chain between the hitch and the drawbar of the trailer. Assume a drawbar pull of P.

(i) For the tractor and trailer with no lift and no drawbar pull; Figure 6.15(a) and (b).

Moments about A for the trailer:

$T\ a = W'\ (a-b)$ where T is the vertical force on the tractor drawbar

$$T = W'\ \frac{a-b}{a}$$

Moments about O for the tractor:

$$V_f\ x + T\ x' = W\ x_r$$

$$V_f = W\frac{x_r}{x} - T\frac{x'}{x} = W_f - T\frac{x'}{x} \qquad (6.27)$$

Substituting for T from above

$$V_f = W_f - W'\frac{(a-b)\ x'}{a\ \ x} \qquad (6.28)$$

Moments about Q for the tractor

$$V_r\ x = W\ x_f + T\ (x + x')$$

$$V_r = W_r + T\frac{x + x'}{x}$$

Substituting for T from above:

$$V_r = W_r + W'\frac{(a-b)(x+x')}{a\ x} = W_r + W'\ \frac{a-b}{a} + W'\frac{(a-b)\ x'}{a\ \ x} \qquad (6.29)$$

The significance of the terms in Equations 6.28 and 6.29 can be identified as follows:

$W'\ \frac{a-b}{a} = T$ - weight from the trailer drawbar

$W'\ \frac{a-b}{a}\ \frac{x'}{x}$ - weight from tractor front wheels due to T *(ii) For the tractor and trailer with pull P and lift force F, Figure 6.15(c)*

Moments about A for the trailer

$$W' \, (a-b) + P\,y' + T\,a = F\,(a-c)$$

where T is vertical force on the tractor drawbar and is now assumed to act downwards on the trailer drawbar.

$$T = F\frac{a-c}{a} - W'\frac{a-b}{a} - P\frac{y'}{a}$$

Moments about Q for the tractor

$$V_r\,x + T\,(x + x') = W\,x_f + P\,y' + F\,(x+x'+c)$$

$$V_r = W_r + P\frac{y'}{x} + F\frac{x+x'+c}{x} - T\frac{x+x'}{x}$$

Substitute for T from above:

$$V_r = W_r + P\frac{y'}{x} + F\frac{x+x'+c}{x} - F\frac{(a-c)(x+x')}{a\,.\,x} + W'\frac{(a-b)(x+x')}{a\,.\,x} + P\frac{y'\,(x+x')}{a\,.\,x}$$

$$V_r = W_r + W'\frac{(a-b)(x+x')}{a\,.\,x} + P\frac{y'(a+x+x')}{a\,.\,x} + F\frac{c(a+x+x')}{a\,.\,x} \qquad (6.30)$$

Problem 6.15

Show that the weight on the front wheels of the tractor with weight transfer hitch is:

$$V_f = W_f - W'\frac{(a-b)x'}{a\,.\,x} - P\frac{(x'+a)y'}{a\,.\,x} - F\frac{c(a + x')}{a\,.\,x} \qquad (6.31)$$

The terms in these equations showing the weight transferred to the rear tractor wheels can be identified as follows:

W_r , W_f , W'	- static weight on the respective wheels
$W'\frac{a-b}{a} = T$	- weight on the trailer drawbar
$W'\frac{a-b}{a}\frac{x'}{x}$	- weight transferred from tractor front wheels due to T
$P\frac{y'}{a}$	- weight from trailer wheels due to P
$P\frac{y'}{x}$	- weight from tractor front wheels due to P
$P\frac{y'}{a}\frac{x'}{x}$	- weight from tractor front wheels due to transfer from trailer wheels
$F\frac{c}{a}$	- weight from trailer wheels due to F
$F\frac{c}{x}$	- weight from tractor front wheels due to F
$F\frac{c}{a}\frac{x'}{x}$	- weight from tractor front wheels due to transfer from trailer wheels

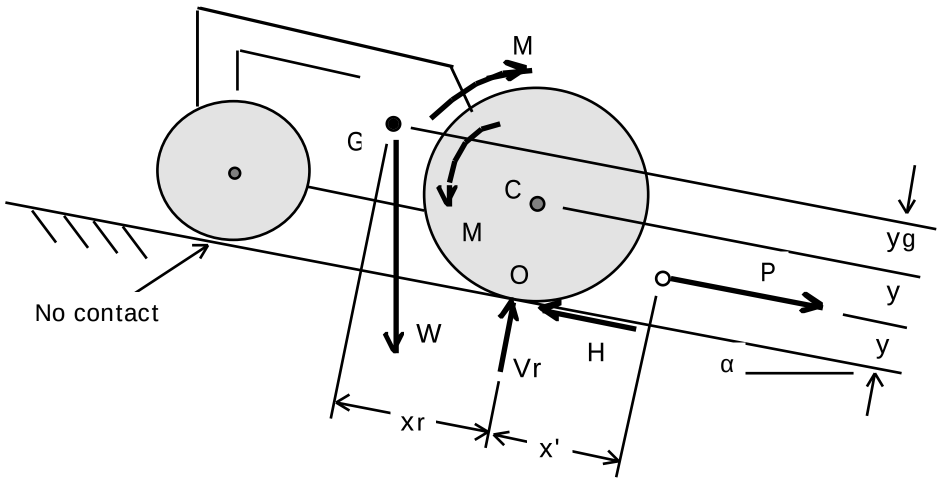


Figure 6.16: Operating parameters for tractor on slope with impending instability

6.5 IMPENDING INSTABILITY

The following analysis, which is similar to that given by Sack (1956), illustrates the factors which limit the operation and performance of the tractor as a result of impending instability in the vertical longitudinal plane.

Consider the four-wheel tractor on the slope with drawbar pull parallel to the ground surface; $\theta = 0$, as shown in Figure 6.16.

For impending instability, ie, $V_f = 0$

Moments about C for the tractor:

$$M + W \sin\alpha\, y_g = W \cos\alpha\, x_r + P\, y$$

Resolve perpendicular to the slope:

$$V_r = W \cos\alpha$$

Resolve parallel to the slope:

$$H = P + W \sin\alpha$$

Take moments above C for wheel:

$$M = H\, r$$

Write

$$H = \psi' V_r$$ where ψ' is the gross tractive coefficient (ie, based on H)

Substitute for H, P and M above:

$$M = \psi W \cos\alpha\; r$$

Substitute for H, P and M above:

$$\psi W \cos\alpha\; r = W \cos\alpha\, x_r + \psi W \cos\alpha\; y - W \sin\alpha\; y - W \sin\alpha\, y_g$$

$$\sin\alpha\,(y + y_g) = \cos\alpha\,(x_r + \psi\,(y - r))$$

$$\tan\alpha\,(y + y_g) = x_r - \psi(r - y)$$

$$\tan\alpha\,(r - y' + y_g) = x_r - \psi(r - y)$$

$$\psi' = \frac{x_r - \tan\alpha\,(r - y' + y_g)}{y'}$$

$$\psi' = \tan\alpha + \frac{x_r - \tan\alpha\,(r + y_g)}{y'} \qquad (6.32)$$

Dividing through by $(r+ y_g)$ gives

$$\psi' = \tan\alpha + \frac{\dfrac{x_r}{r + y_g} - \tan\alpha}{\dfrac{y'}{r + y_g}}$$

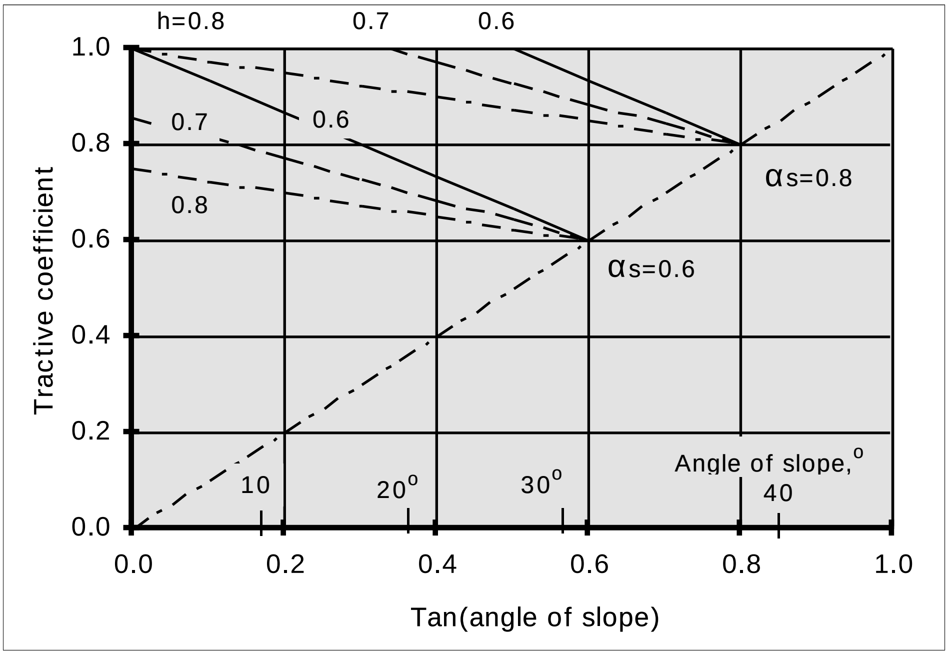


Figure 6.17: Relationships for tractor with impending instability

Write $\dfrac{x_r}{r + y_g} = \tan\alpha \text{ (static)} = \tan\alpha_s$ (6.33)

α_s is the angle of slope that would cause the tractor to tip as a rigid body about the ground contact points under static, ie, no drawbar pull conditions; α_s is usually a large angle, about 40^o for most tractors.

Write $\dfrac{y'}{r + y_g} = \dfrac{\text{drawbar height}}{\text{centre of gravity height}} = h$ (6.34)

A typical value for h is 0.6.

$$\psi' = \tan\alpha + \frac{\tan\alpha_s - \tan\alpha}{h} \quad (6.35)$$

Here ψ is the tractive coefficient that must be achieved to bring the tractor to impending instability when it is operating on a slope α.

(i) If ψ' required to travel up the slope is less than ψ' given by Equation 6.35, then the tractor will not reach impending instability.

(ii) If ψ' required to travel up the slope is greater than ψ' given by Equation 6.35, then the tractor will reach impending instability.

(iii) If ψ' required to travel up the slope is greater than the maximum ψ' possible, then the tractor wheels will slip.

Figure 6.17 shows a plot of ψ versus tan α for various values of:

(i) $\tan\alpha_s = 0.6$ (high centre of gravity) and 0.8(typical centre of gravity)

(ii) h = 0.6 (typical drawbar height), 0.7 and 0.8 (a high and dangerous hitch point).

The region where tanα > ψ' is not feasible; the tractor will slide off the slope.

The example shows a tractor on the slope where tan α = 0.3.

(i) For $\psi'_{max} = 0.8$ (good traction conditions) instability can occur for h = 0.7 or 0.8 because ψ'_{max} is greater than ψ' = 0.72 or 0.67 required.

(ii) For $\psi'_{max} = 0.6$ (moderate traction conditions) instability cannot occur even for h = 0.8 because ψ'_{max} is less than ψ = 0.67 required; the wheels will slip.

The general conclusion to be drawn is that impending instability:

(i) is unlikely to occur with normal drawbar heights, moderate slopes and common traction conditions; usually the wheels slip

(ii) may occur (often with fatal consequences) where traction conditions are good or have been enhanced by the use of strakes (traction aids), where slopes are steep and particularly where the drawbar or the loading point has been raised.

It should also be noted that, while the above simple, static analysis suggests the tractor is relatively safe if used correctly, in practice dynamic effects may influence its behaviour and create dangerous situations. For example, acceleration of the tractor forwards introduces an inertia force through the centre of gravity that has a moment about the rear axle which tends to tip the tractor rearwards. The opposite will be true when the tractor is being braked; here weight is removed from the rear wheels which may adversely affect their braking capacity.

Problem 6.16

What drawbar height will just bring the Farmland tractor to impending instability at the maximum tractive force on a horizontal soil surface? The following additional data apply:

Total area of soil - wheel contact patch	A	= 0.076 sq. m;
Cohesion of soil	c	= 2.0 kPa
Angle of internal friction of soil	φ	= 32^o

From Figure 6.15:

For impending instability: $V_f = 0$

Resolving horizontally: $D = H = A\ c + W \tan\varphi$

Moments about O:

$$H\ y' = W x_r$$

$$H = W \frac{x_r}{y'}$$

$$W \frac{x_r}{y'} = A\ c + W \tan\varphi$$

$$y' = \frac{W x_r}{A\ c + W \tan\varphi}$$

This confirms the conclusion given above that, as the soil becomes stronger, (c and φ increase) the height of the drawbar pull that is required to cause impending instability decreases, ie, instability is more likely under good traction conditions than under poor when the wheels will slip rather than the tractor to tip.

For the Farmland tractor:

$$y' = \frac{W x_r}{(A\ c + W \tan\varphi)} = \frac{2850 \text{ x } 9.81 \text{ x } 0.54}{.076 \text{ x } 2 \text{ x } 2000 + 2850 \text{ x } 9.81 \text{ x } \tan 32} = 0.85 \text{ m}$$

There are places on many tractors at this height to which a load could be attached; it is clearly very dangerous! Loads should always be attached to the standard drawbar.

Problem 6.17

Repeat Problem 6.16 for the tractor: (i) travelling up a slope
(ii) travelling down a slope

6.6 REFERENCES

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