The Mechanics of
Tractor – Implement Performance

Theory and Worked Examples

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CHAPTER 2

TRACTOR MECHANICS

CONTENTS

Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.

CHAPTER 2

TRACTOR MECHANICS

2.1 INTRODUCTION

The tractor is a machine and the application of the general principles of mechanics to it provides a simple but fundamental understanding of its operation and ideal performance. The actual performance will be less than this, and may be much less, mainly because of the losses which occur at the wheel / ground contact surface.

In a similar way to other engineering disciplines, we can define the elements or components of the tractor in terms of general mechanics without needing to know their detailed form. Thus the engine (power source) can be represented in terms of its torque and speed without having to specify its type (thermodynamic or electrical), its operating principle (internal or external combustion), its operating cycle (two or four stroke) or its fuel source (diesel or petrol (gasoline)). Similarly the transmission system can be expressed in terms of the transmission ratio without specifying its form or operating principle (mechanical (gears, chains, belts), hydrostatic (fluid pressure) etc).

> We can thus separate the application of the principles of mechanics to the tractor from the particular forms of the mechanisms that appear in the particular tractor that we see in the laboratory or field.

2.2 IDEAL ANALYSIS (without losses)

Consider a tractor operating on a firm surface as shown in Figure 2.1. Although the tractor is moving, the equations of equilibrium can be applied to it because it is assumed that there is no acceleration.

Consider the engine running at a rotational speed N_e driving the drive wheels without losses through a transmission with an overall ratio of q. As a consequence of the reduction in speed by a factor of 1/q, there is a corresponding increase in torque by a factor of q. These values correspond to the `velocity ratio´ and the `mechanical advantage´ from elementary physics.

2.2.1 Speed analysis

For the tractor as shown in Figure 2.1(a):

Drive wheel diameter = D

Engine speed = N_e

Overall transmission ratio $q = \dfrac{\text{Engine speed } N_e}{\text{Drive wheel speed } N_w}$

Drive wheel rotational speed $N_w = \dfrac{N_e}{q}$

If we assume that there are no losses in motion due to slip between the wheel and the surface:

Travel speed, V_o = Linear speed of wheels

$$= \pi\, D\, N_w$$

$$= \frac{\pi\, D\, N_e}{q} \qquad (2.1)$$

This analysis shows that the travel speed depends directly on the engine speed and inversely on the gear ratio.

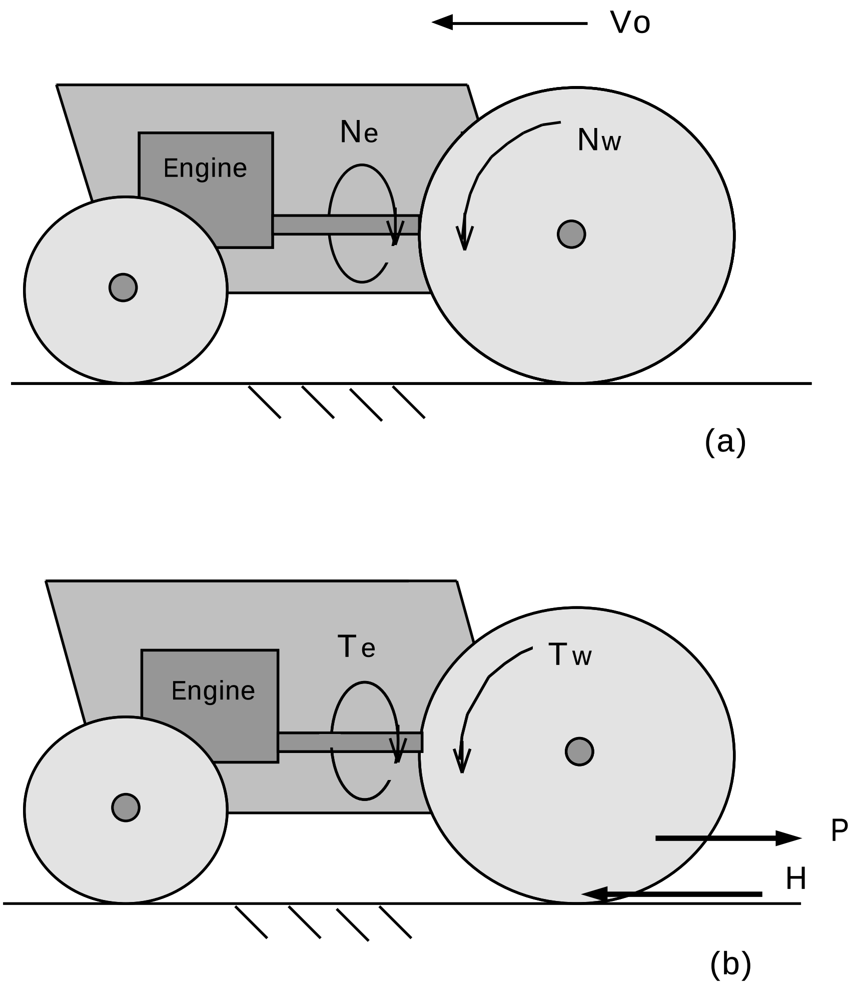


Figure 2.1 Mechanics of the tractor under ideal conditions
(a) Speed analysis; (b) Torque / force analysis

2.2.2 Torque / force analysis

For the tractor as shown in Figure 2.1(b):

Engine torque $= T_e$

Drive wheel torque, $T_w = q\,T_e$

Equilibrium requires that this torque is equal and opposite to the moment of the soil reaction, H on the wheel:

$$H\,\frac{D}{2} = T_w = q\,T_e$$

$$H = \frac{2\,q\,T_e}{D}$$

If we assume that there are no other horizontal external forces acting (such as rolling resistance), equilibrium also requires that:

Drawbar pull, P = Soil reaction, H

$$P = \frac{2\,q\,T_e}{D} \qquad (2.2)$$

This analysis shows that the drawbar pull depends directly on the torque generated by the engine and on the gear ratio. This assumes that the wheel / ground contact can generate the reaction to P.

2.2.3 Power analysis

Engine power, $Q_e = 2\pi\,T_e\,N_e$ (2.3)

Drawbar power, Q_d = Drawbar pull . travel speed

$= P\,.\,V_o$ (2.4)

$$= \frac{2\,q\,T_e}{D}\;.\;\frac{\pi\,D\,N_e}{q}$$

$$= 2\pi\,T_e\,N_e$$

= Engine power

Thus, if we neglect losses in forward motion due to wheelslip and in drawbar pull due to rolling resistance, all of the power from the engine is available at the drawbar.

The above represents the ideal situation which might apply approximately to the tractor working on hard surfaces with small drawbar pulls and small wheelslips.

However, in many agricultural situations, wheelslip is significant, hence the travel speed of the tractor will be less, and may be much less, than the ideal value calculated above. Also, much of the torque on the rear wheels goes to drive the tractor forward against the rolling resistance of both the driving and the rolling wheels. Hence the drawbar pull will be less, and may be much less, than the ideal value calculated above.

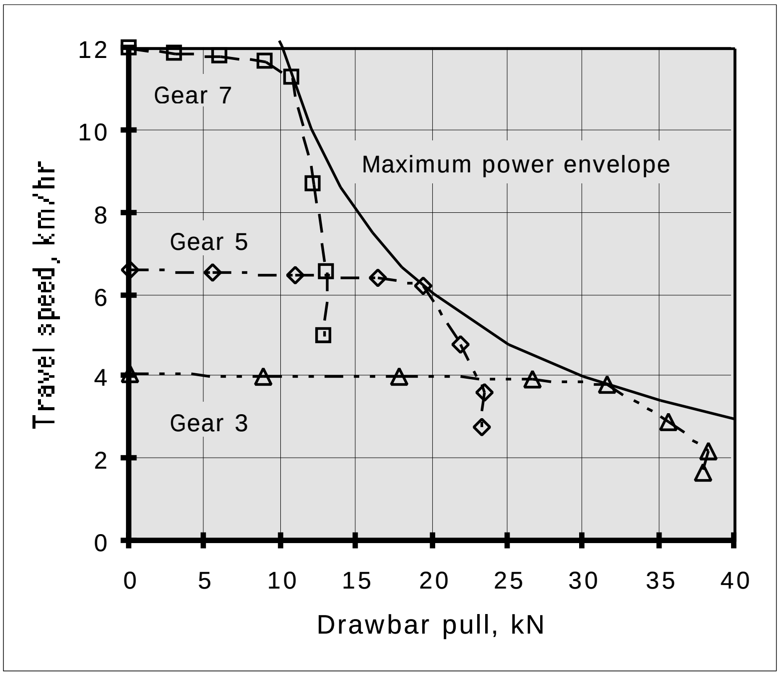
The actual tractive performance of the tractor in various gears on two types of surface, viz., a hard surface (firm, dry soil or road) and a soft surface (cultivated soil), is considered in Chapters 3 and 4, respectively.

Engine		Ideal tractor drawbar performance in gears					
Speed	Torque	Drawbar Pull, kN			Travel speed, km/hr		
rpm	Nm	Gear 3	Gear 5	Gear 7	Gear 3	Gear 5	Gear 7
Gear ratio->		139.5	85.6	47.3	139.5	85.6	47.3
2390	0	0.00	0.00	0.00	4.07	6.63	11.99
2370	40	8.86	5.43	3.00	4.03	6.57	11.89
2350	80	17.71	10.87	6.01	4.00	6.52	11.79
2325	120	26.57	16.30	9.01	3.96	6.45	11.67
2250	142	31.51	19.33	10.68	3.83	6.24	11.29
1730	161	35.65	21.88	12.09	2.94	4.80	8.68
1300	173	38.31	23.51	12.99	2.21	3.61	6.52
1000	171	37.86	23.23	12.84	1.70	2.77	5.02

(a)

Maximum power performance envelope	
Drawbar Pull	Travel Speed
kN	km/hr
6.0	20.10
7.0	17.23
8.0	15.08
10.0	12.06
12.0	10.05
14.0	8.61
16.0	7.54
18.0	6.70
20.0	6.03
25.0	4.82
30.0	4.02
35.0	3.45
40.0	3.02

(b)



(c)

Figure 2.2: Data for (a) ideal performance of Farmland tractor in 3 gears at maximum governor setting; (b) maximum power envelope; (c) plot of these data

Problem 2.1

For a local tractor (of any type):

(a) Measure the transmission ratios in each gear by (securely) raising the drive wheels and either:
    (i) turning the engine by hand and counting revolutions of engine and wheels
    (ii) running the engine and measuring the speed of engine and drive wheels with a tachometer
(b) Check your answers by:
    (i) taking appropriate measurements of the transmission elements - counting gear teeth, measuring pulley or sprocket diameters etc
    (ii) driving the tractor on a hard surface and measuring the travel speed, and rolling radius
    (iii) inspection of the owner's manual or parts book, if available.

2.2.4 Ideal performance graphs

Figure 2.2 shows the torque (Nm) - engine speed (rpm) data from an actual test on the engine from the hypothetical 'Farmland' tractor [1]. It also shows the ideal performance (travel speed (km/hr) versus drawbar pull (kN)) graphs for the Farmland tractor in 3 gears based on the Equations 2.1 and 2.2 and data from Table 1, Appendix I.

The shape of these graphs will be discussed more fully in Chapter 3.

Problem 2.2

Plot similar graphs for the other gears of the Farmland tractor.

2.2.5 Performance envelopes

The graphs shown in Figure 2.2 and others to be plotted in Problem 2.2 give the characteristic graphs for the tractor with discrete gears. Such gears result in `steps´ in the curves defining areas in which the tractor can work and other areas between the steps in which the engine could work but which are unavailable because gears with appropriate ratios are not fitted to the tractor.

If the tractor were fitted with a stepless or infinitely variable transmission, the ratio could be varied to keep the engine operating at maximum power. This would give the (ideal) performance `envelope´ or boundary within which the tractor must work. This is also shown in Figure 2.2 (c) for the constant maximum power of the engine (33.6kW); it is plotted for arbitrarily chosen values of the drawbar pull and calculated travel speeds shown in Figure 2.2(b).

2.2.6 Conclusion

The simple analysis given above suggests that the actual performance of the tractor will reflect the performance of the engine:

(i) travel speed is determined by engine speed
(ii) drawbar pull determines engine torque
(iii) both travel speed and torque also depend on transmission ratio.

Further, the travel speed - drawbar pull performance is limited by the maximum engine power envelope which appears as an hyperbola on the travel speed / drawbar pull graph space.

As shown later in Chapter 3, the actual travel speed - drawbar pull graphs and the corresponding envelope will be different because losses in travel speed due to wheelslip, in drawbar pull due to rolling resistance and in power due to both.

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[1] Test data have been extracted from Australian Tractor Test Report No 78 (Brown and Baillie, 1973). Other numerical data for this tractor, which are used in this book, have been extracted and are presented in Appendix II.

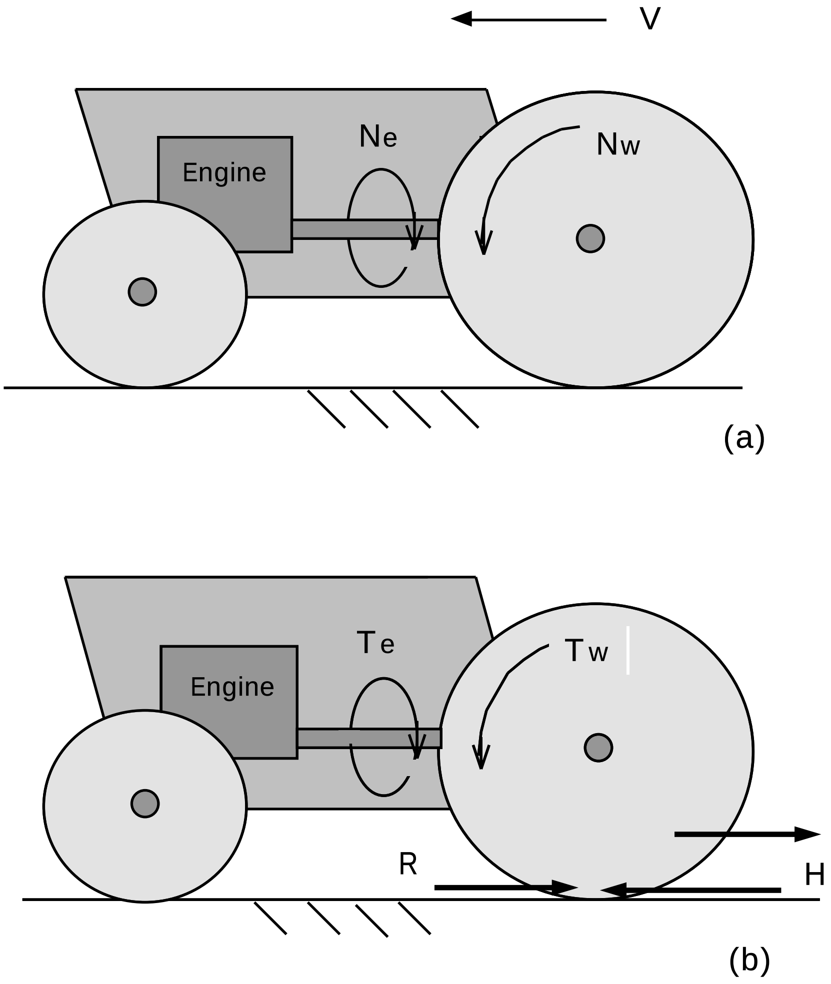

Figure 2.3 Mechanics of the tractor with losses
(a) Speed analysis; (b) Torque / force analysis

2.3 ANALYSIS WITH LOSSES

Consider a tractor again operating on a firm surface as shown in Figure 2.3. Although the tractor is again moving, the equations of equilibrium can be applied to it because it is assumed that there is no acceleration.

2.3.1 Speed analysis

The tractor is now moving with a speed V (less than the ideal travel speed, V_o above), Figure 2.3(a).

We can then define wheelslip as:

$$\text{Wheelslip, } i = \frac{V_o - V}{V_o} \qquad (2.5)$$

Where, V_o = theoretical travel speed (as in Equation 2.1 above)
V = actual travel speed

Substituting for V_o from Equation 2.1

$$V = V_o(1 - i) = \frac{\pi D N_e}{q}(1 - i) \qquad (2.6)$$

2.3.2 Force analysis

A rolling resistance force (R) which is assumed to act horizontally on the wheel at the wheel / ground contact patch, opposes motion of the tractor, Figure 2.3(b).

For equilibrium of the external horizontal forces acting on the tractor:

$$H = P + R \qquad (2.7)$$

2.3.3 Power analysis

Considering power transmission at the wheels.

Output power = Input power - Power loss

ie, Drawbar power = Wheel power - Power loss

Hence, Power loss = Wheel power - Drawbar power

$$= 2\pi T_w N_w - P\,V$$

$$= 2\pi\ H \frac{D}{2}\frac{V_o}{\pi D} - P\,V = H\,V_o - P\,V$$

$$= H\,V_o - (H - R)\,V = H\,(V_o - V) + R\,V$$

$$= H.V_o\ i + R\,V = H\,V_s + R\,V \qquad (2.8)$$

Here V_s is the slip velocity, ie, the velocity of the wheel relative to the surface at the surface / wheel contact.

We can identify the terms in this equation as:

Total power loss = Power loss due to slip + Power loss due to rolling resistance

Minimizing the total power loss thus is matter of minimizing the sum of the loss due to slip and that due to rolling resistance. This is a complex problem when it is realized, for example, that the effect of weight on the driving wheels is to decrease the slip loss and increase rolling resistance loss. This will be discussed further in Chapter 4.

2.4 OTHER MEASURES OF PERFORMANCE

2.4.1 Efficiency

(a) Tractive efficiency

We define tractive efficiency,

$$\eta_t = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Drawbar power}}{\text{Wheel power}}$$

$$= \frac{P.V}{H.V_o} = \frac{(H - R)}{H}(1 - i) \qquad (2.9)$$

$$= (1 - \frac{R}{H})\ (1 - i)$$

$$= \frac{P}{(P + R)}(1 - i) \qquad (2.10)$$

The tractive efficiency that appears here contains two terms:

(i) $\frac{P}{(P+R)}$ which represents a ‘force’ efficiency; thus when there is no rolling resistance (R = 0) this factor in the tractive efficiency = 1.

(ii) (1- i) which represents a ‘speed’ efficiency; again when there is no wheelslip (i = 0), this factor in the tractive efficiency = 1.

It might be thought that the tractive efficiency, which is one of the most important measures of tractor performance, could be determined on the basis of Equation 2.10. However, the major difficulty with this approach is that, in practice, it is not possible to determine a relationship between rolling resistance and slip or, in general, to determine rolling resistance when a wheel is undergoing a slip.

Hence, it is necessary to determine the tractive efficiency by measuring drawbar and wheel power directly by measuring:
(i) drawbar pull, P, with a tension load (force) cell between the tractor and a load vehicle or implement
(ii) travel speed, V, by timing over a known distance
(iii) wheel torque, T_w , with a torque load cell in the transmission to the driving wheels
(iv) wheel speed, N_w , by counting wheel revolutions over a known time period

Then tractive efficiency, $\eta_t = \frac{P\,V}{2\,\pi T_w N_w}$ (2.11)

(b) Transmission efficiency

We can define transmission efficiency:

$$\eta_r = \frac{\text{Power to wheels}}{\text{Power from engine}} = \frac{2\,\pi\,T_w\,N_w}{2\,\pi\,T_e\,N_e} \qquad (2.12)$$

The maximum transmission efficiency is dependent on the design and the quality of the transmission elements. In a geared transmission there is little or no loss in velocity, $N_w = N_e / q$.

Hence any losses are due to a loss in torque; thus $T_w < q\,.\,T_e$

For good quality gears the maximum efficiency is about 98% per pair of gears; hence with, say, 3 pairs of gears in the change transmission and another 2 pairs in the differential / final drive, the maximum efficiency will be (0.98)^5 = 90%. Little improvement in efficiency can be obtained by more accurate or elaborate gearing; other types of transmission will be no more efficient.

(c) Engine efficiency

We can define engine efficiency:

$$\eta_\varepsilon = \frac{\text{Power from engine}}{\text{Power in fuel}} = \frac{2\,\pi T_e\,N_e}{1000\text{ FC C}} \tag{2.13}$$

where FC = fuel consumption rate, kg/min
C = calorific value of the fuel, kJ/kg

The maximum value for engine efficiency is dependent on and strictly limited by the thermodynamics of the engine processes. A maximum value of about 35% for a diesel engine can be expected; other types of engine will, in general, be less efficient.

(d) Overall efficiency

We can also define the overall efficiency for the tractor:

$$\eta_o = \frac{\text{Drawbar power}}{\text{Fuel power}}$$

$$= \frac{\text{Engine power}}{\text{Fuel power}} \cdot \frac{\text{Wheel power}}{\text{Engine power}} \cdot \frac{\text{Drawbar power}}{\text{Wheel power}}$$

$$= \text{Engine efficiency} \cdot \text{Transmission efficiency} \cdot \text{Tractive efficiency}$$

$$= \eta_\tau \cdot \eta_\rho \cdot \eta_\varepsilon \tag{2.14}$$

Consider typical maximum values for these variables:

$$\eta_o = 0.3 \times 0.90 \times 0.75$$

$$= 20\ \%$$

Because the maximum tractive efficiency is low and highly variable and the other efficiencies are high (transmission) or strictly limited (engine), any significant increase in the overall efficiency of tractor performance will be achieved by increasing the tractive efficiency. Research into an understanding of the traction process and into more efficient traction devices is directed to this end.

2.4.2 Tractive coefficient (pull - weight ratio)

As will be shown later, the performance of a tractor depends to a significant degree on its weight and, in particular, on the weight on the driving wheels. It is therefore useful to define a non-dimensional drawbar pull - weight ratio termed:

$$\text{Tractive coefficient, } \psi = \frac{\text{Drawbar pull}}{\text{Weight on driving wheels}} \qquad (2.15)$$

The tractive coefficient is a number which characterizes the interaction between the wheel and the surface in an analogous way to which coefficient of (sliding) friction characterizes the interaction between one body sliding on another. Where a different wheel and surface may be considered similar to those for which the tractive coefficient is known, then for the same wheelslip:

Drawbar pull = Tractive coefficient x weight on wheel

Where a tractor operates on a slope the tractive coefficient should logically be based on the total force parallel to the ground, ie, on the drawbar pull plus the component of the weight of the tractor down the slope.

Where a four-wheel tractor is considered, and with other tractors also, the weight used may be the total weight on all wheels. In quoting values of tractive coefficient, it is therefore necessary to state which weight has been used.

Problem 2.3

Estimate the maximum pull - (total) weight ratio for some local traction devices, eg, tractor, locomotive, draught animal or human.

2.5 SUMMARY

TRACTOR PERFORMANCE PARAMETERS

Parameter	**Engine**	**Transmission**	**Wheels**
Input force	Combustion pressure	Engine torque, T_e	Wheel torque, T_w
* Force conversion ratio	Variable with rotation	Gear ratio, q	Force radius
* Theoretical force	-	Engine torque x gear ratio	Tractive force, H
* Force losses	Mechanical friction	Mechanical friction	Rolling resistance, R
Output force	Engine torque, T_e	Wheel torque, T_w	Drawbar pull, P
Input velocity	Piston velocity	Engine speed, N_e	Wheel speed, N_w
* Velocity conversion ratio	Variable with rotation	Gear ratio, q	Rolling radius
* Theoretical output velocity	Engine speed, N_e	Engine speed / gear ratio	Wheel linear speed, V_o
* Velocity losses	Nil	Nil	Wheelslip, i
Output velocity	Engine speed, N_e	Wheel speed, N_w	Travel speed, V
Input power	Fuel power	Engine power, Q_e	Wheel power, Q_w
* Theoretical output power	-	-	Tractive power, Q_t
Output power	Engine power, Q_e	Wheel power, Q_w	Drawbar power, Q_d
Input/output efficiency	Fuel efficiency, η_f	Transmission efficiency,η_r	Tractive efficiency, η_t

Table 2.1 Summary of tractor performance parameters (Parkhill, Pers. comm)

2.6 REFERENCES

Brown, W.T. and Baillie, W.F. (1973) *Australian Tractor Test Report No 78, Leyland 253.* Australian Tractor Testing Committee, University of Melbourne.

Parkhill, J. G., Personal communication